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Teacher Training Module: Mathematics Learning Cycle Twelve

Highest Common Factor (HCF) and Least Common Multiples (LCM)

Sindh Technical Assistance –
Development through
Enhanced Education Programme
(STA-DEEP)



THE AGA KHAN UNIVERSITY

School Education & Literacy Department (SE&LD)

Government of Sindh.

Introduction and Rationale of the Training

Dear Teachers!

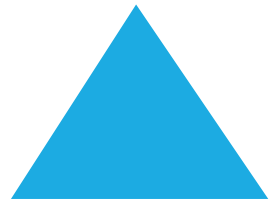
Welcome to the new phase of the Continuous Professional Development (CPD) Program. In the previous phase, we had focused on pedagogical skills that helped you to develop your skills to make classroom more interactive, participative, and joyful for our students. In the new phase, we will continue practicing those pedagogical skills and also learn about the introduced content knowledge and skills in Mathematics, Science, English, Urdu, and Sindhi. As a result, you will be better prepared to deal classroom situation using modern teaching strategies integrated with subject knowledge.

Our vision

Our common goal is to improve the quality of teaching in schools all over Sindh. We want students to become active and collaborative learners, problem solvers, and critical thinkers who approach tasks with creativity and confidence. They are conceptually clear about the subject content and have the skills to link this content with the world around them. To make this possible, we, as teachers, must be better prepared for the classroom demands in pedagogy and the subject content. Moreover, we aim to professionalize these trainings so that the CPD teacher training courses make an impact and substantially change student performance.

Our Teaching Philosophy

The CPD training sessions, including this training, follow a participatory teaching philosophy that engages teachers to apply and practice active and collaborative learning, as well as engage in self and peer reflection to become community of practice. The objective is not only to improve the teaching practices but to help you understand the theory of the subject content and the strategies that help students apply the content in daily life with confidence and mastery.



Supporting You

The training module is designed to support you in your classroom teaching. It will introduce you to the subject content and some approaches for use in the classroom. This will make your teaching more manageable and help you grow as a skillful teacher.

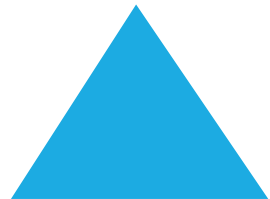
Acknowledgement

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Highest Common Factor (HCF) and Least Common Multiples (LCM)

Learning Objectives: By the end of the session, the teachers will be able to:



Explore factors as the number of the equal-sized group and size of group using concrete material;



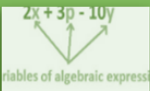
Find the highest common factor (HCF) using manipulatives;



Calculate the Least Common multiples (LCM) by using common multiples method;



Exploring relationship between HCF and LCM;







Solve real life problems of LCM and HCF.



Session Plan

Instructional strategies/activities

Time	Objective/purpose of the activity	Activities/learning experiences	Materials/resources
 10 mins	Activity 1:  To assess prior knowledge of factors and multiples	Warm-up Activity: - Look at the numbers provided in each square box and place it in the relevant column (refer to Handout-12.1) - Collect random responses and conclude the activity	Handout-12.1
 30 mins	Activity 2:  Exploring Factors and Highest Common Factors using Concrete Material	Group Work: - Divide the teachers into 4 groups - Give 18 paper plates and cutouts of stars, squares, and rectangles to each group. - Carry out the following two activities Activity 2A: - Ask teachers to take 12 stars and form as many possible equal-sized groups as they can using paper plates such that paper plates show the	Paper plates 74 Cutouts: <ul style="list-style-type: none"> • Stars 48 • Rectangles 72 • Squares 60 Record sheet A and B Handout-12.2 Handout 12.3 A Handout 12.3 B

		<p>number of groups and the number of stars in each plate is considered as the size of the group</p> <ul style="list-style-type: none">- Record the number of possible equal-sized groups on the record sheet in handout-12.2- Collect random responses and record on the board- Repeat it for 15 squares- Repeat it for 18 rectangles- Ask teachers to observe Record sheet 'A' and share their observations- Discuss factors as a number that divides another number exactly leaving no remainder and connect the concept with the possible number of equal-sized groups and the size of each group formed in the activity. <p>Activity 2B:</p> <ul style="list-style-type: none">- Ask teachers to take 12 stars, 15 squares, and 18 rectangles- Instruct teachers that using all three cutouts of shapes form as many possible equal groups as they can on provided paper plates such that paper plates show the number of groups- Record the number of possible equal-sized groups and the size of each shape in each group in Record sheet 'B'	
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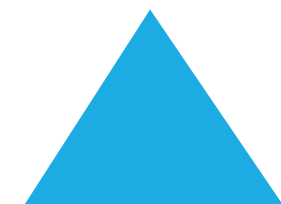


- Ask teachers to observe the record sheet and share their observations
- Discuss common factors and the highest common factors. Connect the idea with the common and highest common number of equal-sized groups and the size of the groups.

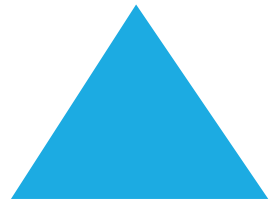
Facilitator's Notes:



- The factor of a number is a divisor that divides the given number completely without leaving any remainder. For instance, in Activity 1, the number of possible equal-sized groups and the size of the groups show the factors of each number of given cutouts.
 - A common factor is a factor that is shared by multiple numbers. For example, in Activity 2B, the number of common equal-sized groups shows the common factors of the numbers 12, 15, and 18.
 - The highest common factor (HCF) is the largest whole number that is shared by given numbers. For instance, the largest common equal-sized group number in Activity 2B shows the HCF of the numbers 12, 15, and 18.
- Other terms used to refer to the highest common factor include the greatest common factor (GCF).



		<p>Individual Work:</p> <ul style="list-style-type: none">- Ask teachers to do the activity given in Handout-12.2 <p>Activity 2C:</p> <ul style="list-style-type: none">-Write the numbers 10 to 20 on the chalkboard.- Ask teachers to do the tasks given in Handout-12.3A- Once teachers complete ask the following follow-up questions: <p>Additional questions:</p> <ol style="list-style-type: none">1. Which numbers have the fewest number of factors? [Answer: 11, 13, 17, 19]2. Which numbers have only a factor of 1 and itself? [Answer: 11, 13, 17, 19]3. What are the numbers called which have only one as a factor and the number itself as the second factor? [Prime numbers]4. What are prime numbers? [A prime number is a natural number greater than 1, which is only divisible by 1 and itself.]5. Do even numbers always have even factors? [No]6. Do odd numbers always have odd factors? [Yes]	
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		<p>-After discussing follow-up questions writes the following two division sentences on the board</p> $24 \div 6 =$ $25 \div 6 =$ <p>- Ask teachers the following question:</p> <ol style="list-style-type: none">1. Can we divide 25 by 6?2. Is this division different from the division in the first problem? <p>- Collect random responses and conclude the task.</p> <p>Facilitators' Notes:</p> <ul style="list-style-type: none">• Factors can also be understood from the division lens for example, 6 is a factor of 24 because we can divide 24 by 6, with no remainder.• Factors of a number are either smaller than or equal to the number itself whereas.• Both problems $24 \div 6$ and $25 \div 6$ differed fundamentally because there was no remainder (or a remainder of 0) in the first problem and a non-zero remainder in the second problem.• In other words, 6 is a factor of 24, and 24 is a multiple of 6. But 6 is NOT a factor of 25 and 25 is NOT a multiple of 6.	
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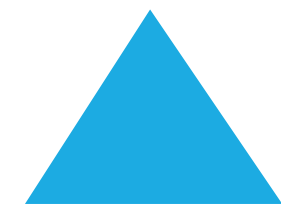




		<ul style="list-style-type: none"> • While both 24 and 25 can be divided by 6, we will say that only 24 is divisible by 6 and 6 is a factor of 24. <ul style="list-style-type: none"> - Ask teachers to do the activity given in Handout-12.3B - Collect random responses and conclude the activity. 	
 15 mins	<p>Activity 3:</p>  <p>“Solve real-life problems involving HCF”</p>	<p>Group Activity 3A:</p> <ul style="list-style-type: none"> - Divide teachers into groups of 4 - Ask teachers to read and discuss the given problems in Handout-12.4 and find the various ways in which the problem can be solved - Collect random responses and elaborate on the practical use of HCF in the context of real-life <p>Group Activity 3B:</p> <ul style="list-style-type: none"> - Ask teachers to refer to STB textbooks of Grades 3, 4, 5, and 6 and discuss the progression of HCF in each Grade level - Ask teachers to refer to the problem sums given in the STB Grade 5 and 6 textbooks and solve at least 5 HCF problems - Ask each group to present one problem and its solutions to the whole class. <p>Facilitator’s Notes:</p> <ul style="list-style-type: none"> - The question should challenge teachers to identify the highest common factor, as this is essential for forming the largest possible 	Handout-12.4 STB Textbooks (Grades 3,4,5 and 6)



		<p>equal-sized groups across both sections. This exploration aims to develop an understanding of the HCF and highlight its significance in real-world applications.</p> <ul style="list-style-type: none"> - Discuss the broader implications of HCF and its use in real-life contexts, such as splitting items into smaller sections, equally distributing two or more sets of items into their largest groupings, and arranging objects into rows or groups. 	
 15 mins	<p>Activity 4:</p>  <p>“Developing understanding of LCM”</p>	<p>Activity 4:</p> <ul style="list-style-type: none"> - Inform the teachers that we are going to play a game " Fizz Buzz " - Ask the teachers to form a circle around the facilitator and follow the instructions: -When the facilitator starts counting from 1 then the teacher next to the facilitator will say next count and similarly, the next teacher says next count and continues counting until the circle is complete. - While calling your count when you reach a number that is a multiple of 3, say "Fizz" instead of the number. - While calling your count when you reach a number that is a multiple of 5, say "Buzz." - While calling your count when you reach a number that is a common multiple of both 3 and 5, say "Fizz Buzz." 	Handout-12.5

		<ul style="list-style-type: none"> - While playing the game, anyone who says a number instead of "Fizz" or "Buzz" will be out of the circle. - Do one practice round before starting the game formally. - Whoever remains in the circle until the end will be the Winner! - Conclude the activity and discuss the multiples and common multiples. <p>Facilitator's Notes:</p> <ul style="list-style-type: none"> • A multiple is a product that we get when one number is multiplied by another number. • In the activity, the numbers 15 and 30 are numbers that appear in the "times table" of both numbers. We can continue and find many more common multiples. <ul style="list-style-type: none"> - Ask teachers to do the task given in Handout-12.5 - Collect random responses and conclude the task. 	
 30 mins	<p>Activity 5:</p>  <p>Solve real-life problems involving LCM</p>	<p>Group Activity 5A:</p> <ul style="list-style-type: none"> - Divide teachers into groups of 4 - Ask teachers to read and discuss the given problems in Handout 12.6 and find the various ways in which the problems can be solved - Collect random responses and discuss the problem 	<p>Handout 12.6 STB Textbooks (Grade 3,4,5 and 6)</p>

		<p>- Elaborate on the practical use of LCM in the context of real-life. For instance, LCM helps in planning recurring events efficiently. It is useful for purchasing items to maintain an adequate supply when restocking periods vary. Additionally, LCM assists in synchronizing schedules and predicting future occurrences based on multiple repeating cycles.</p> <p>Group Activity 5B:</p> <ul style="list-style-type: none">- Ask teachers to refer to STB textbooks of Grades 3, 4, 5, and 6 and discuss the progression of LCM in each Grade level- Ask teachers to refer to the problem sums given in the STB Grade 5 and 6 textbooks and solve at least 5 LCM problems- Ask each group to present one problem and its solutions to the whole class. <p>Facilitator's Notes:</p> <ul style="list-style-type: none">- The least of the common multiples is the one that is often of the greatest interest to us because of its mathematical as well as practical utility.	
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 <p>10 mins</p>	<p>Activity 6:</p>  <p>Relationship between HCF and LCM</p>	<p>- Write "10×11" on the board and pose questions in the following sequence:</p> <ol style="list-style-type: none">1. What is the product of 10 and 11? [110; Write the response on the board]2. What is the LCM of 10 and 11? [110; Write the LCM on the board]3. What is the HCF of 10 and 11? [1; Write the HCF on the board] <p>- Ask the teachers if they observe any relationship between the HCF, LCM, and the product of 10 and 11.</p> <p>- Expected answer: "Yes, if we multiply the HCF, which is 1, and the LCM, which is 110, we get the product of 10 and 11, which is 110."</p> <p>- Encourage teachers to try with five different pairs of numbers to see if this relationship holds true for other numbers.</p> <p>- Collect random responses and conclude the activity by highlighting that the product of two numbers is equal to the product of their LCM and HCF.</p> <p>Facilitator's Notes:</p> <p>-The relation between HCF and LCM is that the product of 2 numbers is equal to the product of the HCF of two numbers and the product of LCM of the two numbers. For instance, if 'a' and 'b' are two numbers.</p>	Handout 12.7
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		<p>The formula that expresses the relationship between their LCM and HCF is given as:</p> $\text{LCM}(a,b) \times \text{HCF}(a,b) = a \times b$ <p>- Ask teachers to do the task given in Handout-12.7</p>	
 15 mins	<p>Activity 7:</p>  Assessment	<p>-Teachers will attempt the following assessments</p> <p>1) Which number below is the highest common factor of the numbers 24 and 36?</p> <p>A. 3 B. 6 C. 12</p> <p>2. How many factors does a Prime number have?</p> <p>A. 1 B. 2 C. 0</p> <p>3. Find the least number, which is exactly divisible by 5, 10, 20</p> <p>A. 80 B. 100 C. 120</p>	

4) Ali's friend asked him to bring the same number of chocolates and biscuits. The store sells chocolates in packs of 50, biscuits in packs of 20. Find out how many chocolates Ali will buy from the store.

5) Rachel has 24 red candies and Maya has 18 green candies. They want to arrange the candies in such a way that each row contains an equal number of candies and each row should have only red candies or green candies. What is the greatest number of candies that can be arranged in each row?

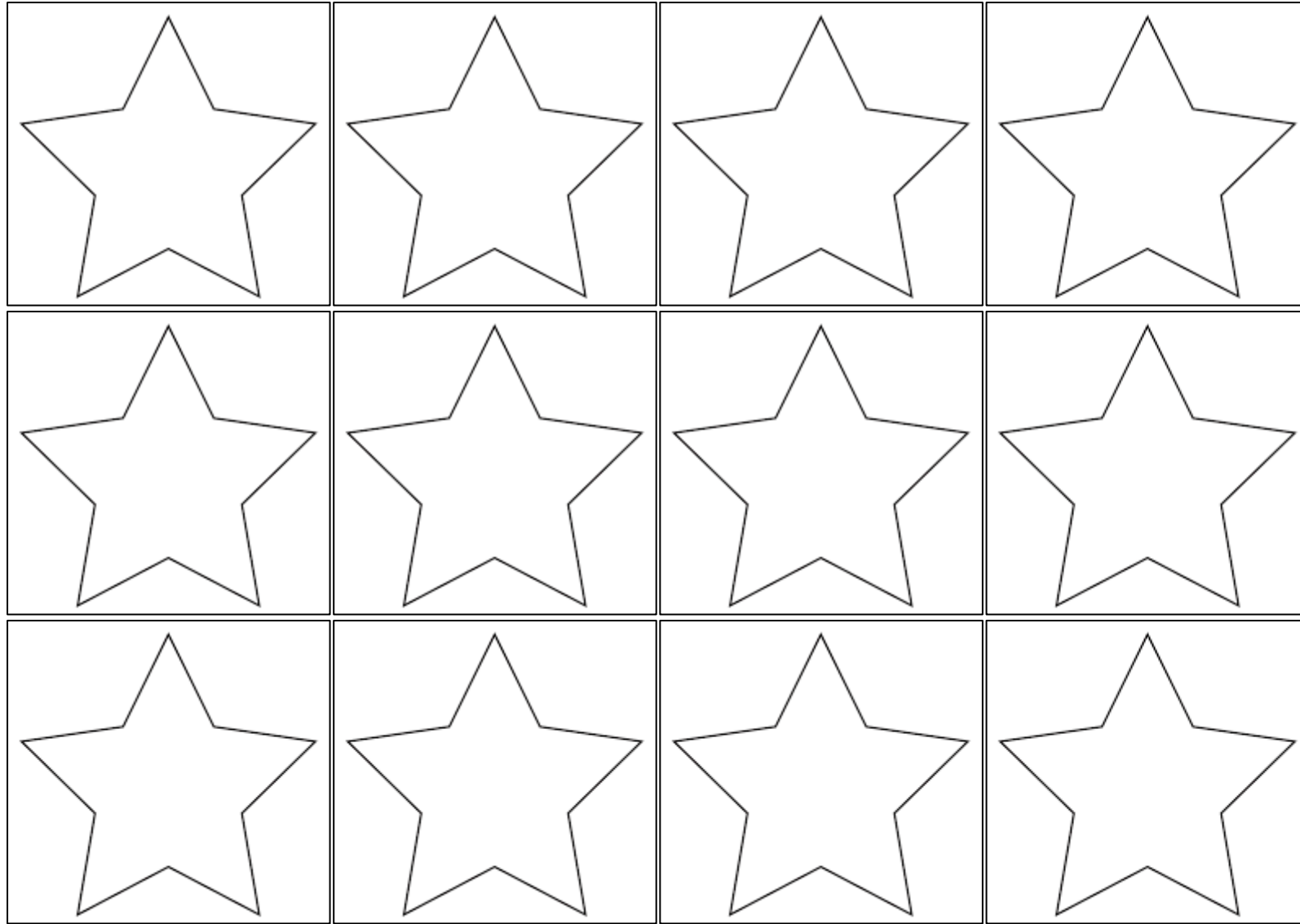


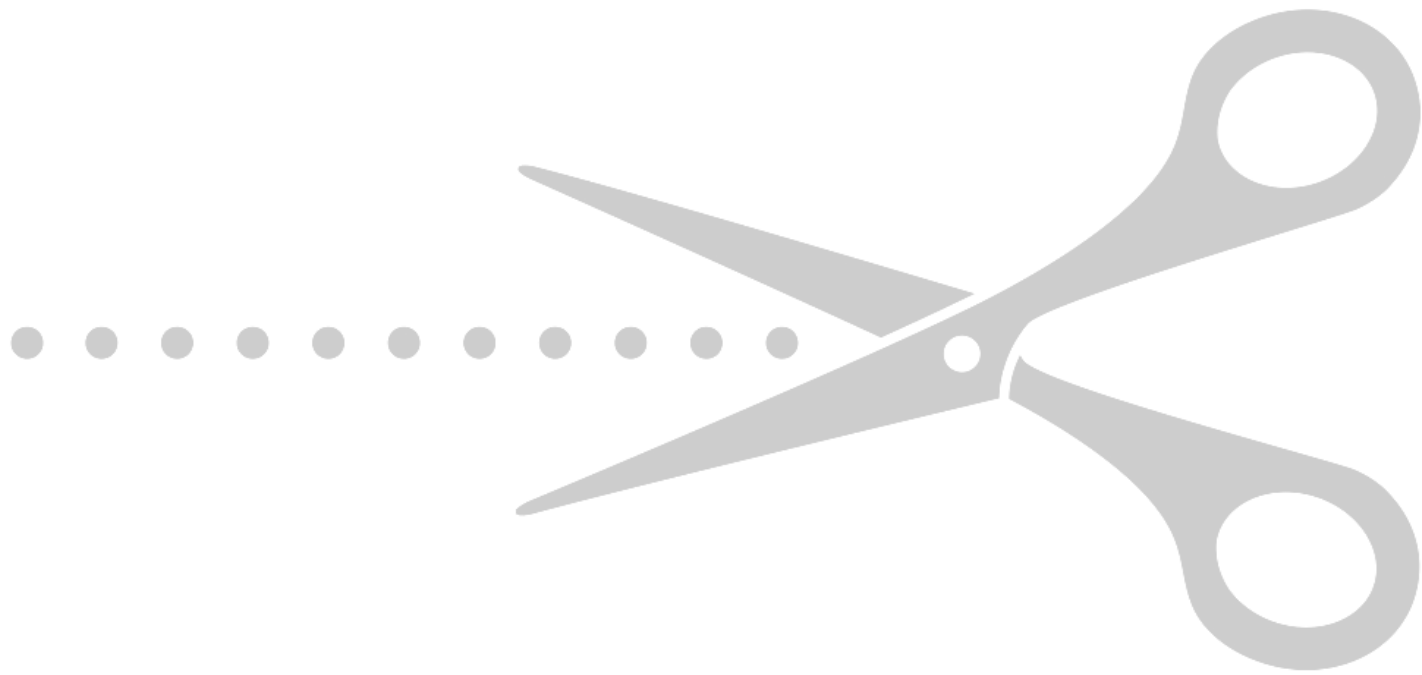
Handout 12.1

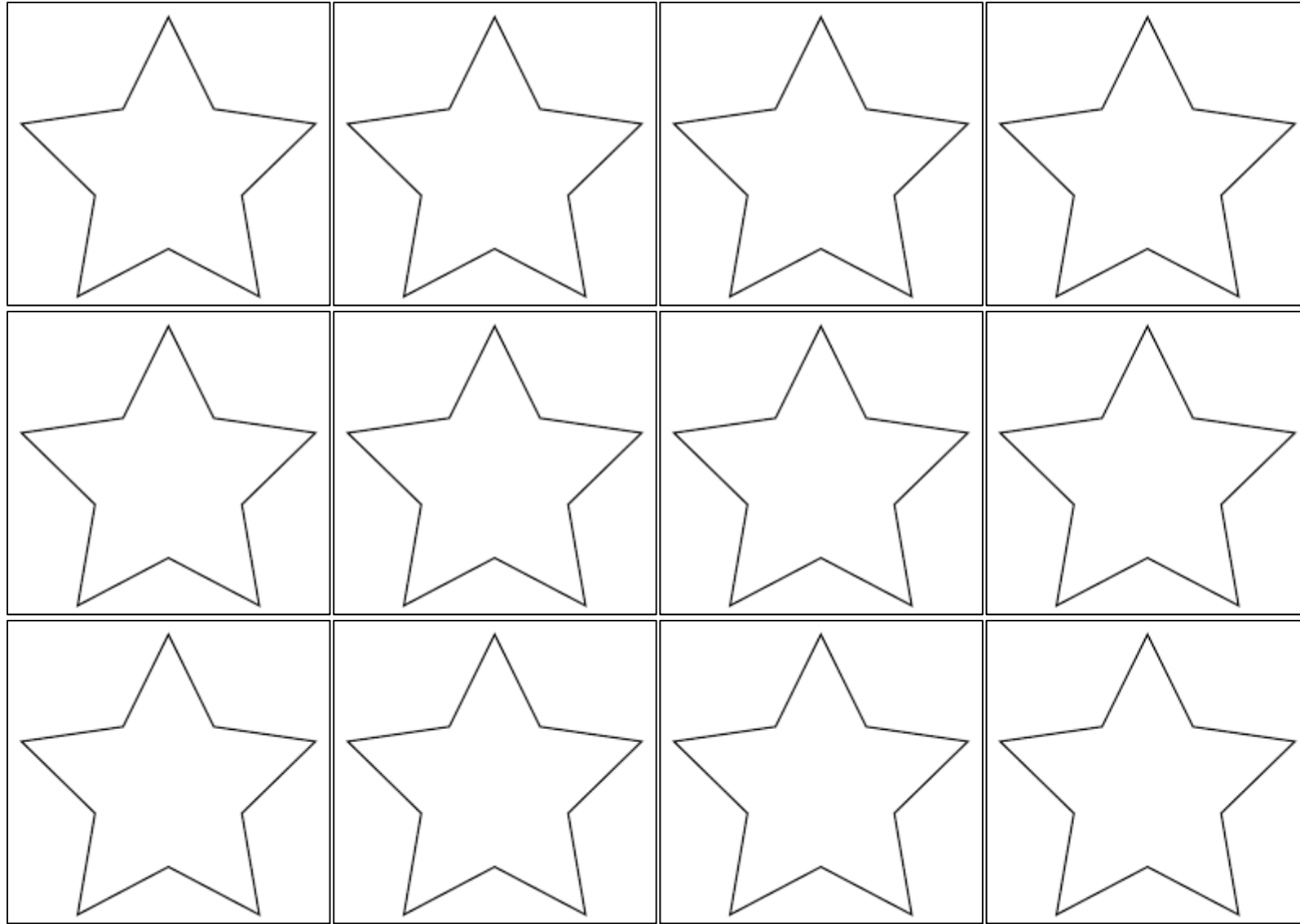
Instructions: Look at the numbers given in the boxes below. Place each number to the correct column

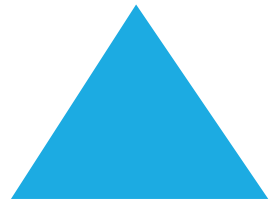
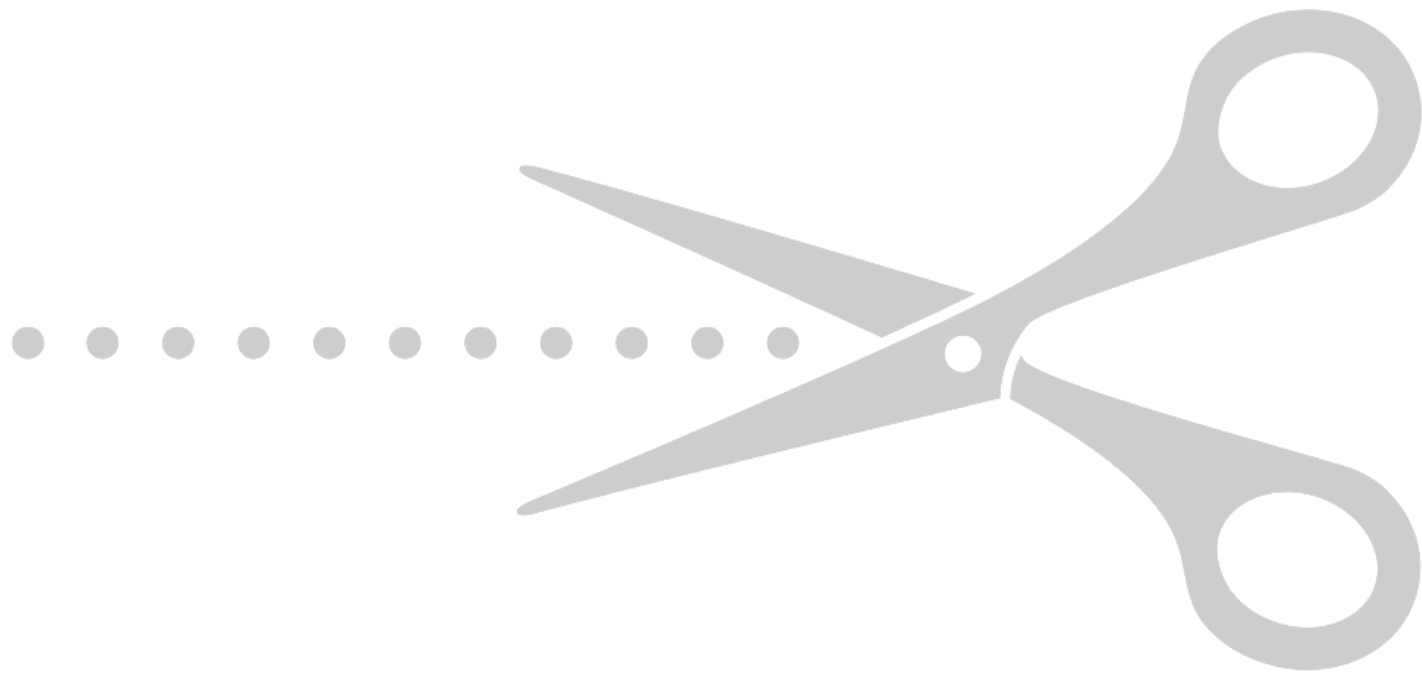
Factors of 12	Multiples of 12	Neither factor or multiple of 12

3	24	6	92	1	2
4	7	60	72	12	17
8	144	48	2	34	40

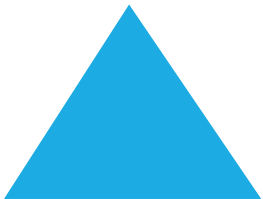
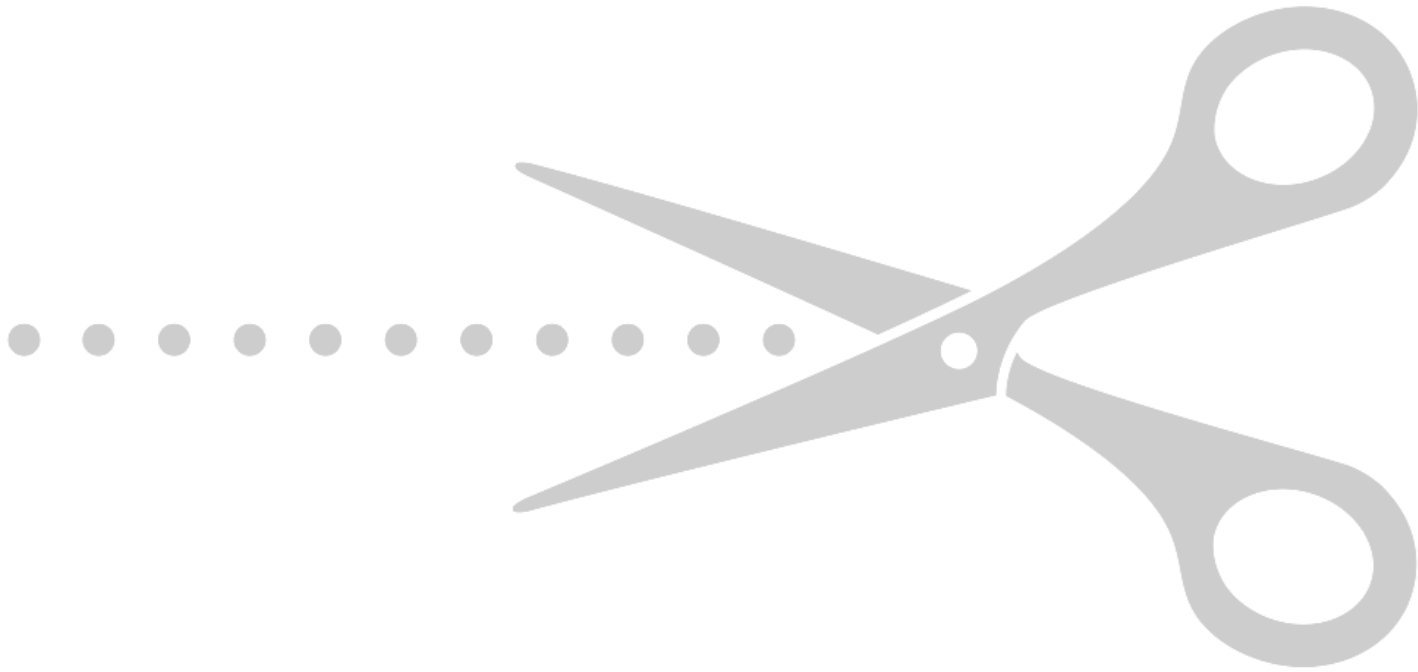




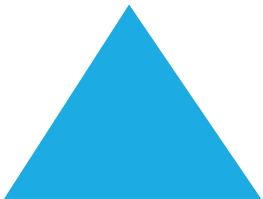
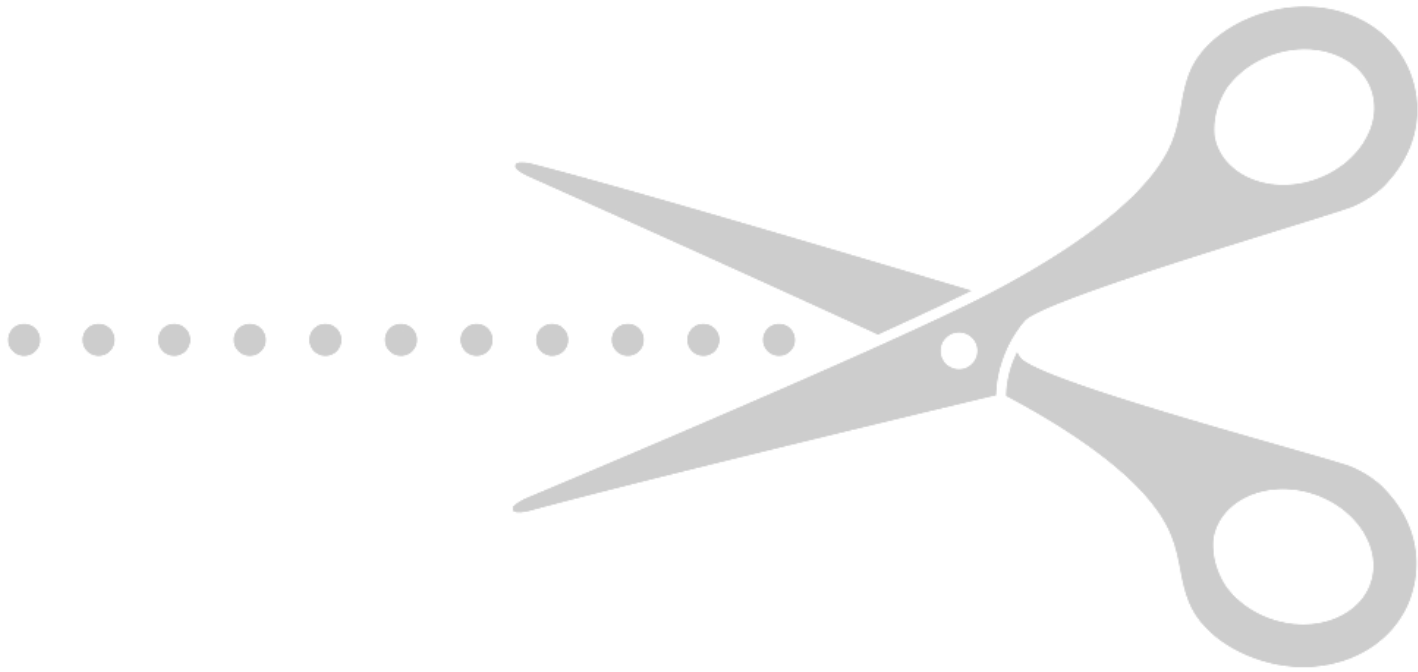




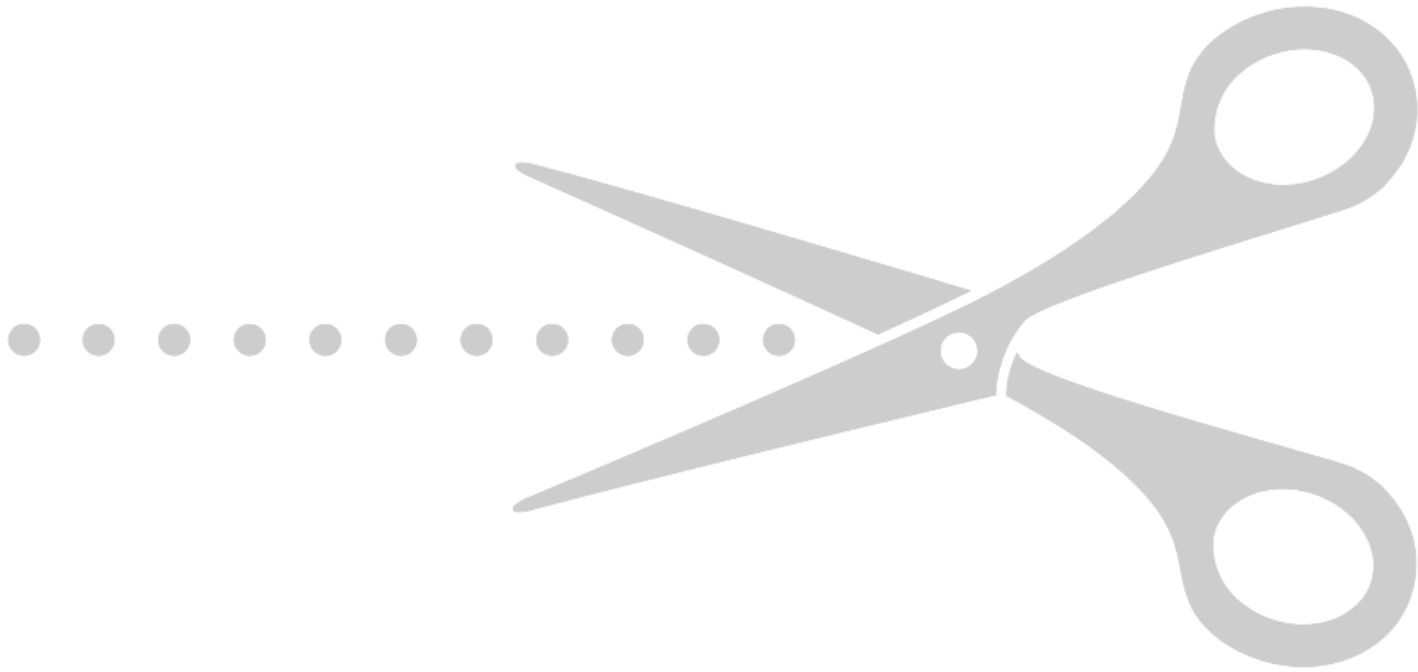




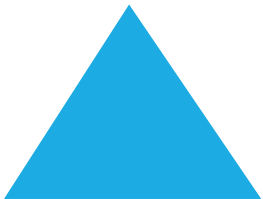
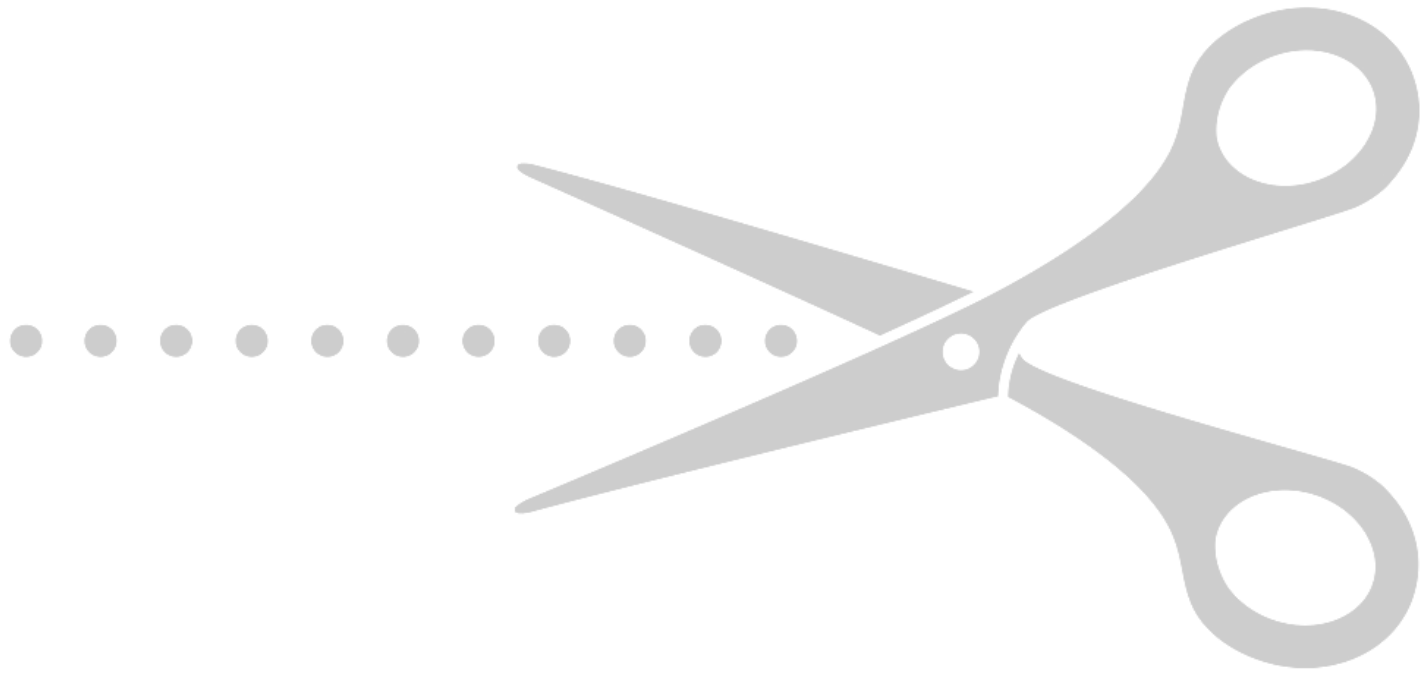













Record Sheet A

Instructions: Record your observations

- 1) Number of equal-sized groups of 12 stars are _____
- 2) Sizes in each group of 12 stars are _____
- 3) Number of equal-sized groups of 15 squares are _____
- 4) Sizes in each group of 15 squares are _____
- 5) Number of equal-sized groups of 18 rectangles are _____
- 6) Sizes in each group of rectangles are _____

Record Sheet B

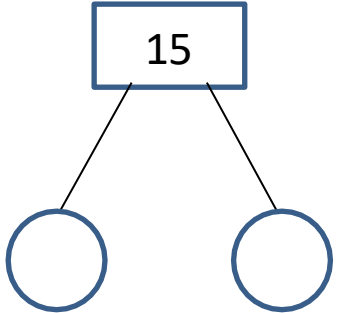
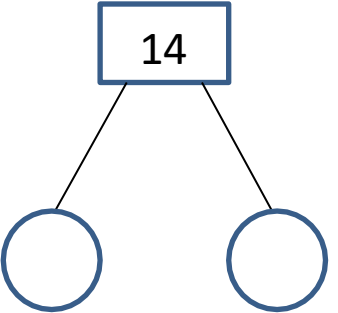
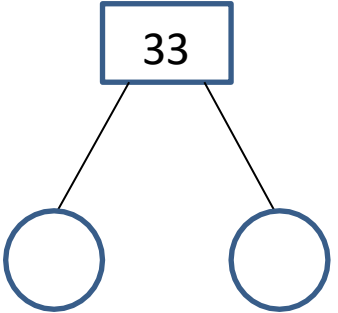
Instructions: Record your observations

- 1) Number of equal-sized groups formed by 12 stars, 15 squares, and 18 rectangles are _____
 - 2) Sizes in each group formed by 12 stars, 15 squares, and 18 rectangles are _____
- 

Handout 12.2

Factor Tree

Instructions: Fill in the missing numbers in these prime factor trees with the prime factors in the circles. Then complete the prime factorization product underneath.

1)  $15 = _ \times _$	2)  $14 = _ \times _$	3)  $33 = _ \times _$
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4)

$12 = _ \times _ \times _$

5)

$30 = _ \times _ \times _$


6)

$27 = _ \times _ \times _$



Handout 12.3 B

Instructions: Read the following statements and write yes or no against each. Also, share reasons of your answers.

- 1) Is 27 is completely divisible by 9? Yes/No
 - 2) Can we say 9 is a factor of 27? Yes/No
 - 3) Is 28 is completely divisible by 8? Yes/No
 - 4) Can we say 8 is a factor of 28? Yes/No
 - 5) Is 15 is completely divisible by 4? Yes/No
 - 6) Can we say 4 is a factor of 15? Yes/No
 - 7) Is 35 is completely divisible by 7? Yes/No
 - 8) Can we say 7 is a factor of 35? Yes/No
 - 9) Is 20 is completely divisible by 3? Yes/No
 - 10) Can we say 3 is a factor of 20? Yes/No
- 

Handout 12.4

Problems:

- 1) For a dinner party, Ali is creating individual servings of starters. He has 24 carrot sticks and 18 celery sticks. If he wants each serving to be identical, with no food left over, what is the greatest number of servings Ali can create?
- 2) Two wires are 12 m and 16 m long. The wires are to be cut into pieces of equal length. Find the maximum length of each piece.

Handout 12.5

Instructions: Find the least common multiples of the following numbers.

1)	LCM (10, 15) =	
2)	LCM (20, 5) =	
3)	LCM (12, 8) =	
4)	LCM (6, 15) =	
5)	LCM (4, 11) =	
6)	LCM (7, 12) =	

Handout 12.6

Problem:

1) On a jogging track, Jahangir completes the track in 30 minutes while Zehra completes the track in 25 minutes.

If they both start at the same time, after how much time they will be side by side together?

2) Two types of ice cream are available in packs of 46 and 24, respectively. Aleena wants to buy the same number of ice-creams of both types. What is the least number of packs of each type of ice cream that she will need to buy?

3) Three bells ring at intervals of 85, 35, and 15 seconds, respectively. If the bells rang together at 8 o'clock, when would they ring together again?

Handout 12.7

Show that:

a) The $\text{LCM}(6, 15) \times \text{HCF}(6, 15) = \text{Product of}(6, 15)$

b) The $\text{LCM}(8, 12) \times \text{HCF}(8, 12) = \text{Product of}(8, 12)$

c) The $\text{LCM}(9, 10) \times \text{HCF}(9, 10) = \text{Product of}(9, 10)$



References:

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2018). Elementary and middle school mathematics_ teaching developmentally (10th ed.). Pearson.



For reference:

List of 1-20 LCs topics

Learning Cycles (LCs)	Topics
LC-1	Developing Number Sense
LC-2	Fractions
LC-3	Decimal and Percentage
LC-4	Ratio and Proportion
LC-5	Introduction to Algebra
LC-6	Algebraic Identities
LC-7	Angle and its Constructions
LC-8	Area and Perimeter
LC-9	Three Dimensional Shapes
LC-10	Information Handling
LC-11	Place Value
LC-12	Highest Common Factor (HCF) and Least Common Multiple (LCM)
LC-13	Fraction Addition and Subtraction
LC-14	Fraction Multiplication
LC-15	Laws of Exponents
LC-16	Square Roots
LC-17	Simultaneous Linear Equations
LC-18	Unit Conversion
LC-19	Pythagoras Theorem
LC-20	Construction of Different Types of Triangles

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