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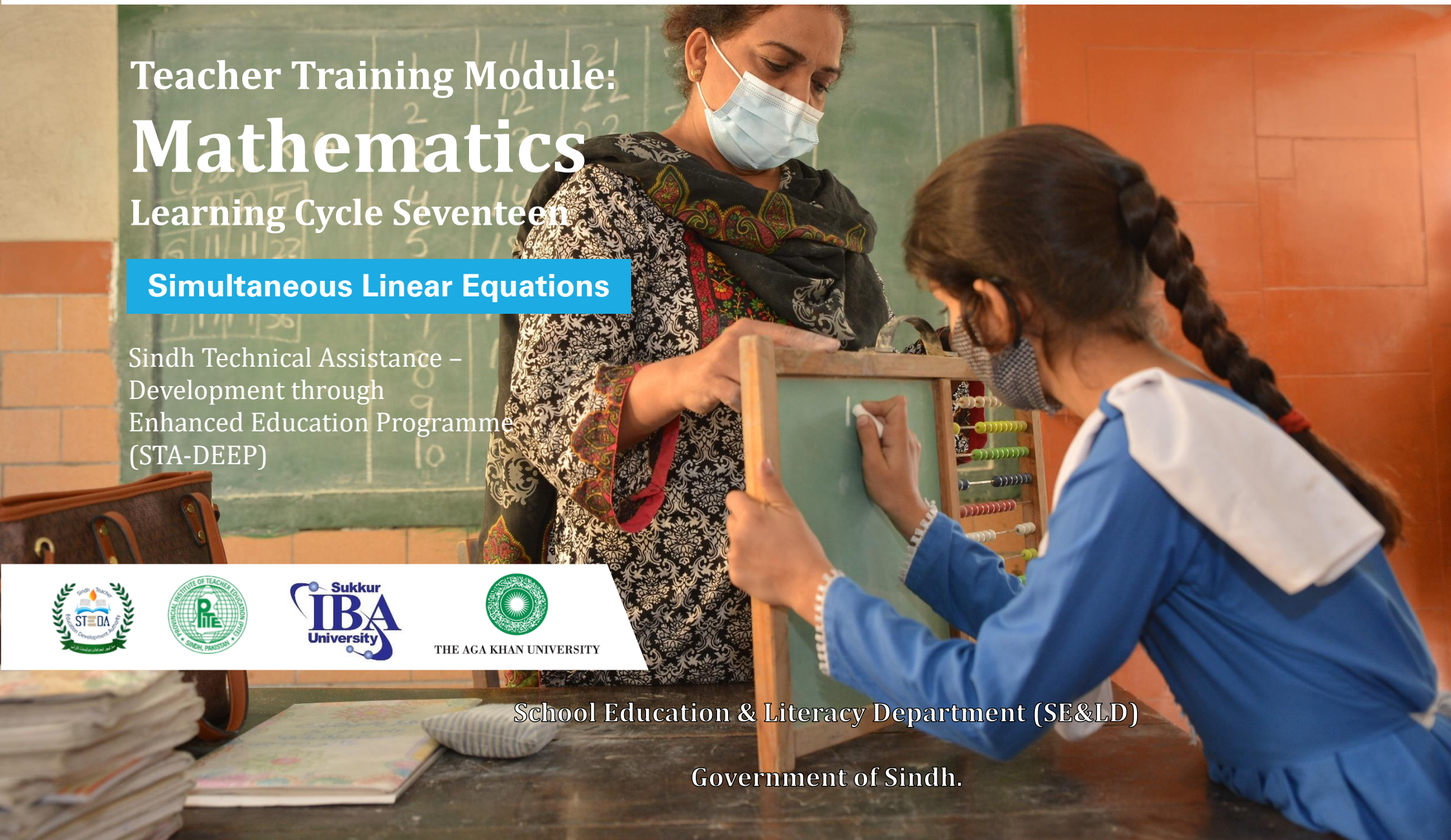
Teacher Training Module:

Mathematics

Learning Cycle Seventeen

Simultaneous Linear Equations

Sindh Technical Assistance – Development through Enhanced Education Programme (STA-DEEP)



School Education & Literacy Department (SE&LD)

Government of Sindh.

Introduction and Rationale of the Training

Dear Teachers!

Welcome to the new phase of the Continuous Professional Development (CPD) Program. In the previous phase, we had focused on pedagogical skills that helped you to develop your skills to make classroom more interactive, participative, and joyful for our students. In the new phase, we will continue practicing those pedagogical skills and also learn about the introduced content knowledge and skills in Mathematics, Science, English, Urdu, and Sindhi. As a result, you will be better prepared to deal classroom situation using modern teaching strategies integrated with subject knowledge.

Our vision

Our common goal is to improve the quality of teaching in schools all over Sindh. We want students to become active and collaborative learners, problem solvers, and critical thinkers who approach tasks with creativity and confidence. They are conceptually clear about the subject content and have the skills to link this content with the world around them. To make this possible, we, as teachers, must be better prepared for the classroom demands in pedagogy and the subject content. Moreover, we aim to professionalize these trainings so that the CPD teacher training courses make an impact and substantially change student performance.

Our Teaching Philosophy

The CPD training sessions, including this training, follow a participatory teaching philosophy that engages teachers to apply and practice active and collaborative learning, as well as engage in self and peer reflection to become community of practice. The objective is not only to improve the teaching practices but to help you understand the theory of the subject content and the strategies that help students apply the content in daily life with confidence and mastery.

Supporting You

The training module is designed to support you in your classroom teaching. It will introduce you to the subject content and some approaches for use in the classroom. This will make your teaching more manageable and help you grow as a skillful teacher.

Acknowledgement

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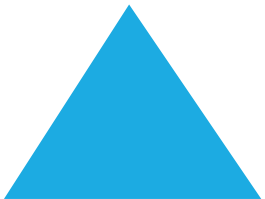
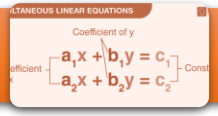
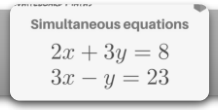


Simultaneous Linear Equations

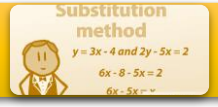
Learning Objectives: By the end of the session, the teachers will be able to:



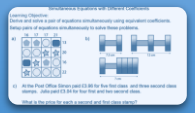
Recognize simultaneous linear equations in two variables;



Read word problems and write simultanous equation;



Solve simultaneous linear equations using methods such as equating coefficients, elimination by substitution, and cross-multiplication;



Solve real life problems using simultaneous linear equations in two variables.

Session Plan

Instructional strategies/activities

Time	Objective/purpose of the activity	Activities/learning experiences	Materials/resources
 15 min	**Activity 1:**  **Assess prior Knowledge of teachers**	**Activity 1:** -The facilitator will write the following linear equations on the board and will ask teachers to guess and verify any three solutions (if possible) in the following linear equations *a)* $x+y= 10$ *b)* $2x+6=10$ *c)* $x-y=2$ *d)* $2x+y=16$ - Collect random responses and conclude the activity by discussing the difference in linear equations in one variable and two variables. **Facilitator's Notes:** **-** Linear equations are equations in which the highest power of the variable is one. - The standard form of a linear equation in one variable is of the form - ax + b = 0. Here, x is a variable, a is a coefficient and b is constant.	Handout-17.1

		- The standard form of a linear equation in two variables is of the form ax + by = c. Here, x and y are variables, a and b are coefficients and c is a constant. **Activity 2:** - Divide teachers into small groups (3 or 4 in each group) - Instruct teachers to fill in the table given in handout-17.1 by guessing values for x and y from the cards such that values satisfy both equations 1 and 2. - Encourage teachers to try multiple pairs of values. - Ask each group to share one example from their table where they found a pair of values that satisfy both equations. - Write these examples on the whiteboard and introduce the concept of simultaneous linear equations **Facilitator Notes:** - Simultaneous linear equations into two variables are a system of two linear equations in two variables that are solved together to find a common solution. It can be represented as follows:	

		Coefficient of y Coefficient of x: $\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$ Constant Here, a_1 and a_2 are the coefficients of x, and b_1 and b_2 are the coefficients of y, and c_1 and c_2 are constant. -The solution for the system of linear equations is the ordered pair (x, y), which satisfies the given equations.	
 15 min	**Activity 2:**  **Forming Simultaneous Linear Equations**	**Activity:** -The facilitator will divide the teachers into small groups and assign each group, two word problems from the Handout 17.2. Each group will read the assigned problems and write the corresponding simultaneous equations. - Collect random responses and ensure each group understands the statements and how to translate them into simultaneous equations - Conclude the activity while highlighting the use of simultaneous linear equations in a real-life context. **Facilitator's Notes:**	Handout 17.2

		- Simultaneous linear equations in two variables involve two unknown quantities to represent real-life problems. It helps in establishing a relationship between quantities, prices, speed, time, distance, etc. resulting in a better understanding of the problems and their possible solutions. - Simultaneous equations are often used when we have at least two unknown quantities and at least two pieces of information involving these quantities. Our first step is to define variables to represent these quantities. The next step is to translate the pieces of information into equations that satisfy the conditions given problem or statement. Once the equations are formed, solve equations simultaneously using different methods such as the substitution method, elimination method, cross multiplication method, or graphical method.	
 45 min	**Activity 3:**  **Solving Simultaneous Linear Equations**	-The facilitator will use first word problem included in the handout 17.3 for solving simultaneous equation using substitution method. -The facilitator will explain methods of solving simultaneous linear equations in the following steps: **Step 1: Problem Statement** The problem gives us two pieces of information: a) The price of the book is 50 rupees more than the notebook.	Handout 17.3 Grade-8 STB Textbook = 5

		b) The total price of the book and the notebook is 115 rupees. **Step 2: Define Variables:** Let *'x'* be the price of the book in rupees and *'y'* be the price of the notebook in rupees **Step 3: Form Equations** We can write these statements as equations: *1)* $x = y+50$ which is $x-y = 50$ *2)* $x + y=115$ So, the pair of simultaneous linear equations is: $x - y = 50$ $x + y=115$ **Step 4: Solve Equations: Substitution Method** **1) Arrange the First Equation:** Consider the following pair of linear equations: $x - y = 50$----------*(i)* $x + y = 115$--------*(ii)* let's arrange the first equation to express 'x' in terms of y as follows $x - y + y = 50 + y \Rightarrow x = 50 + y$ **2) Substitute in the Second Equation:**	

		This expression for x can now be substituted in the second equation, so that we will be left with an equation in 'y' alone. $x + y = 115$ $(50 + y) + y = 115$ $50 + 2y = 115$ $2y = 115 - 50$ $2y = 65$ $y = 32.5$ **3) Find the Value of x:** Once we have the value of y, we can put this back into any of the two equations to find out x. Let's put it into the first equation: $x - y = 50$ $x - 32.5 = 50$ $x = 50 + 32.5$ $x = 82.5$ **4) Final Solution:** The price of the book is Rs 82.5 and the price of the notebook is Rs 32.5 -The facilitator will pose the question, "Why is this process called substitution?"	

		- Collect random responses and discuss that in the substitution method, we express one variable in terms of another using one of the pairs of equations and substitute that expression into the second equation. Hence, it is known as the substitution method. **Practice Activity:** - Ask teachers to solve the second statement given in Handout 3 using the same method i.e., Substitution Method - Collect random responses and conclude the activity. -The facilitator will refer to the third problem statement and help teachers understand **Elimination method of solving simultaneous** linear equations in the following steps: **Step 1: Problem Statement** The problem gives us two pieces of information: 1) Twice the age of the son added to the age of the father equals 56. 2) Twice the age of the father added to the age of the son equals 82. **Step 2: Define Variables:** Let 'x' be the age of son and 'y' be the age of father **Step 3: Form Equations**	

		We can write these statements as equations: $2x + y = 56$ $x + 2y = 82$ So, the pair of simultaneous linear equations is: $2x + y = 56$ $x + 2y = 82$ **Step 4: Solve Equations** - Facilitator will refer to the elimination method to solve the equation **Elimination Method:** Consider the following pair of linear equations: $2x + y = 56$----------*(i)* $x + 2y = 82$--------*(ii)* The coefficients of x in the two equations are 2 and 1 respectively. Let us multiply the first equation by 1 and the second equation by 2, so that the coefficients of x in the two equations become equal. $1 \times (2x + y = 56)$ $2 \times (x + 2y = 82)$ $\Rightarrow \quad 2x + y = 56$ $2x + 4y = 164$	

		Now, let us subtract the two equations, which means that we subtract the left-hand side the two equations and the right-hand side of the two equations, and the equality will still be preserved. We get, $2x + y = 56$ $\pm 2x \pm 4y = \pm 164$ $3y = 108$ $y = \frac{108}{3}$ $y = 36$ Note how x gets eliminated, and we are left with an equation in 'y' alone. Once we have the value of y, we put this value of 'y' into any of the two equations. Let us put this into the first equation: $2x + 36 = 56$ $2x = 56 - 36$ $2x = 20$ $x = \frac{20}{2}$ $x = 10$ Thus, the solution is $x = 10$ and $y = 36$ **Solution: Age of Son is 10 years and age of father is 36 years** -The facilitator will pose the question, "Why is this process called elimination?"	

		-The facilitator will collect random responses and discuss that in the elimination method, we eliminate one variable to find the value of the other variable. This is done by adding or subtracting the equations to cancel out one of the variables, hence it is known as the elimination method. **Practice Activity:** - Ask teachers to solve the fourth statement given in Handout 3 using the Elimination Method. - Collect random responses and conclude the activity. -The facilitator will refer to the fifth statement given in handout 3 and guide teachers to **Cross-multiplication method** to solve simultaneous linear equations in the following steps: **Step 1: Problem Statement** The problem gives us two pieces of information: 1. The difference between the two numbers is 7 2. Two times the smaller number added to the larger number gives 22. **Step 2: Define Variables:** Let 'x' be the first number and 'y' be the second number **Step 3: Form Equations**	

We can write these statements as equations:

1. $x - y = 7$
2. $x + 2y = 22$

So, the pair of simultaneous linear equations is:

$$x - y = 7$$

$$x + 2y = 22$$

Step 4: Solve Equations

- Facilitator will refer to the cross-multiplication method to solve the equation (Grade VIII textbook p. 108)

So, $x = 2$ and y [illegible]

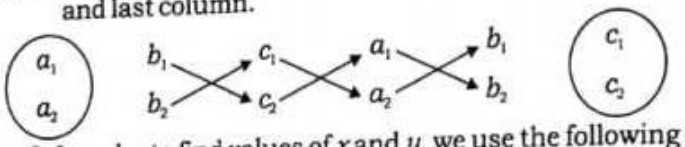
(3) Method of cross-multiplication

In order to explain this method, we take two simultaneous equations in standard form, i.e., $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$. Following are the steps of this method.

Step 1: We write coefficients of variables and constants in the following manner:

$$\begin{array}{cccccc} a_1 & b_1 & c_1 & a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 & a_2 & b_2 & c_2 \end{array}$$

Step 2: We cross multiply in the following way after excluding first and last column.



Step 3: In order to find values of x and y, we use the following equal ratios.

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

Consider the following equations

$x - y = 7$--------- *(i)*

$x + 2y = 22$------*(ii)*

Now, write equations in standard form which is ax+bx+c=0 so,

$x - y - 7 = 0$--------- *(i)*

$x + 2y - 22 = 0$------*(ii)*

By method of cross multiplication,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{b_2a_1 - b_1a_2}$$

Substitute the values in the above equation;

$$\Rightarrow \frac{x}{-22(-1)-(-7)(2)} = \frac{y}{-7(1)-(1)(-22)} = \frac{1}{1(2)-(-1)(1)}$$

$$\Rightarrow \frac{x}{22+14} = \frac{y}{-7+22} = \frac{1}{2+1}$$

$$\Rightarrow \frac{x}{36} = \frac{y}{15} = \frac{1}{3}$$

$$\Rightarrow \frac{x}{36} = \frac{1}{3}$$

$$=> x = \frac{36}{3}$$

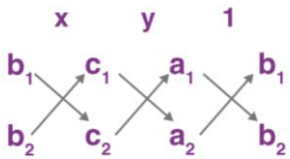
$$=> x = 12$$

$$\Rightarrow \frac{y}{15} = \frac{1}{3}$$

$$=> y = \frac{15}{3}$$

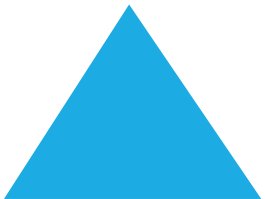
$$=> y = 5$$

		$\Rightarrow x = 12, y = 5$ **Solution: First number is 12 and the second number is 5** **Practice Activity:** - Ask teachers to solve any question 1 part (i) given in Grade-8 STB using the same method. - Collect random responses and conclude the activity. **Facilitator's Notes:** - Methods taught for solving simultaneous linear equations at the elementary level **1) Substitution Method** - Follow the below steps to solve the system of simultaneous linear equations using the substitution method: 1. Rearrange one of the given equations to express x in terms of y. 2. Now, the expression for x can be substituted in the other equation to find the value of y. 3. Finally, substitute the value of y in any of the equations to find the value of x. **2) Elimination Method** - Follow the below steps to solve the system of simultaneous linear equations using the elimination method	

		1. Multiply the given equations by a constant, to make the coefficients of the variables in the equations equal. 2. Add or subtract the equations to eliminate the variable having the same coefficients. 3. Now, solve the equation for one variable. 4. Substitute the variable value in any of the equations to find the value of the other variable. 5. Finally, the ordered pair (x, y) is the solution of the simultaneous equation. **Cross Multiplication Method** - Cross-multiplication is the quickest way to solve a set of linear equations. The figure below as it's easy to remember the cross-multiplication $a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$  - Write the given simultaneous linear Equation on the board $2x + 3y = 15$	

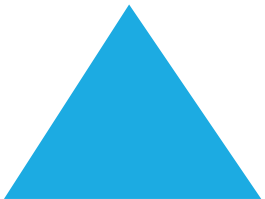
		$4x - 3y = 3$ - Ask the teachers which method they would choose among the following methods and why. A) Elimination Method B) Substitution Method C) Cross-Multiplication Method - Collect random responses and conclude the activity by summarizing that in cases where the coefficients of any of the two variables are already the same (or can be made the same with simple multiplication), the elimination method is often easier to use.	
 30 min	**Activity 4:**  **Solve real-life problems involving Simultaneous Linear Equations**	- Divide teachers into five groups **Activity 4A: Story Creation** - Provide the teachers with the following simultaneous linear equation $x + 2y = 6$ $x - y = 3$ - Instruct the teachers to develop a real-life story or scenario that incorporates the above equation. - Ask teachers to share their stories and solutions	STB Grade 8 Math Textbooks = 5

		- Facilitate a discussion where teachers can reflect on the activity and discuss how this approach can broaden their understanding and enhance their teaching strategies. **Facilitator's Notes:** - Incorporating storytelling and real-life problem-solving into teaching mathematics can broaden students' understanding in several ways: • **Contextual Application:** By creating stories around equations, students learn to see mathematical concepts in various real-world contexts. This helps them understand how abstract mathematical ideas apply to everyday situations, making the concepts more relatable and comprehensible. • **Enhanced Engagement:** Storytelling can make learning more engaging and interesting. Teachers who use this approach may find it easier to capture their students' attention and foster a more dynamic learning environment. • **Critical Thinking:** Developing stories requires students to think creatively and critically about how mathematical equations function in real-life scenarios. This deepens their conceptual understanding and enhances their problem-solving skills.	

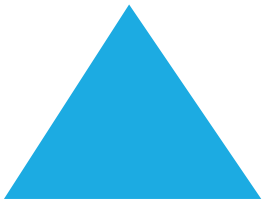


		• **Multiple Perspectives:** Collaboratively creating and discussing stories allows students to see different perspectives and approaches to the same problem. • **Improved Communication:** Translating mathematical equations into stories helps students develop better communication skills. They learn to explain complex concepts in simpler terms, which is crucial for effective understanding. **Activity 4B: Alignment with the textbook** - Ask teachers to refer to the problem sums given in the STB Grade 8 textbooks chapter 6, Exercise # 6.6, and solve all the given problems - Ask each group to present one problem and its solutions to the whole class.	
 15 min	**Activity 5:**  **Assessment**	-Teachers will attempt the following assessments 1) Given the simultaneous linear equations: $x - y = 11$ $4x + y = 14$ What are the values of x and y that satisfy both equations? $A)\ x = 5, y = -6$	

		B) $x = 5, y = 6$ *C)* $x = 6, y = 5$ 2) A linear equation in two variables is of the form ax + by + c = 0, where, *A) a = 0, c = 0* *B)* $a \neq 0, b = 0$ *C)* $a \neq 0, b \neq 0$ 3) For the given simultaneous linear Equation $3x - y = 12$ $2x + y = 13$ Which of the following is the correct value of y? *A) 5* *B) 3* *C) 1* For the given simultaneous linear Equation $2x + 3y = 15$ $4x - 3y = 3$ 4) The sum weights of Faraz and Zehra are 60 kg, and the difference is 2 kg. Find the weights of Faraz and Zehra.	



		5) The sum of the two numbers is 10, and their difference is 6. Make a pair of equations and solve them simultaneously to find the numbers.	



Handout 17.1

Instructions: Fill in the table given in handout-1 by guessing values for x and y from cards. The condition is that the guessed values must satisfy both Equation 1 and Equation 2 in each set.

Set	Equation 1	Equation 2	x	y
1	x + y = 5	x - y = 1		
2	3x + 7y = 27	5x + 2y = 16		
3	4x + 3y = 2	5x + 2y = -1		
4	x - y = 3	x + y = 5		
5	2x + y = 10	3x + y = 14		

x = 2; y = 3

x = 4; y = 2

x = 4; y = 1

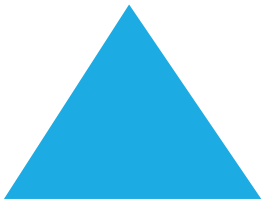
x = 3; y = 2

x = -1; y = 2

Handout 17. 2

Instructions: Read the word problems and write simultaneous equations.

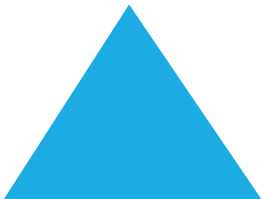
1) The price of a book is Rs 50 more than a notebook and their total price is Rs 115
2) The sum of two numbers is 27 and difference is 17
3) If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82.
4) Two pens and one eraser cost Rs. 35 and 3 pens and four erasers cost Rs. 65.
5) The difference between the two numbers is 7. Two times the smaller number added to the larger number gives 22.
6) If 2 is added to the numerator and denominator it becomes $\frac{9}{10}$ and if 3 is subtracted from the numerator and denominator it becomes $\frac{4}{5}$.



Handout 17. 3

Instructions: Solve the simultaneous equations

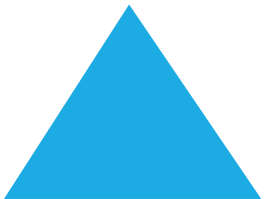
1) The price of a book is Rs 50 more than a notebook and their total price is Rs 115. Find the cost of a book and a notebook.
2) The sum of two numbers is 27 and difference is 17. Find the two numbers.
3) If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82. Find the age of son and father.
4) Two pens and one eraser cost Rs. 35 and 3 pens and four erasers cost Rs. 65. Find the cost of pen and eraser.
5) The difference between the two numbers is 7. Two times the smaller number added to the larger number gives 22. Find the two numbers.
6) If 2 is added to the numerator and denominator it becomes $\frac{9}{10}$ and if 3 is subtracted from the numerator and denominator it becomes $\frac{4}{5}$. Find the fractions.



For reference:

List of 1-20 LCs topics

Learning Cycles (LCs)	Topics
LC-1	Developing Number Sense
LC-2	Fractions
LC-3	Decimal and Percentage
LC-4	Ratio and Proportion
LC-5	Introduction to Algebra
LC-6	Algebraic Identities
LC-7	Angle and its Constructions
LC-8	Area and Perimeter
LC-9	Three Dimensional Shapes
LC-10	Information Handling
LC-11	Place Value
LC-12	Highest Common Factor (HCF) and Least Common Multiple (LCM)
LC-13	Fraction Addition and Subtraction
LC-14	Fraction Multiplication
LC-15	Laws of Exponents
LC-16	Square Roots
LC-17	**Simultaneous Linear Equations**
LC-18	Unit Conversion
LC-19	Pythagoras Theorem
LC-20	Construction of Different Types of Triangles



For reference:

List of Resource Items for LCs (11-20)

Items	No. of items	LC-11	LC-12	LC-13	LC-14	LC-15	LC-16	LC-17	LC-18	LC-19	LC-20
Dice	8	✓							✓		
Pair of scissors	8	✓	✓								
Paper Plate	18		✓								
Red beans	½ kg	✓	✓								
Counters (Red/Black)	10								✓		
Counters (Blue/White)	10								✓		
Color pencil box	4			✓	✓						
Measuring tape	8								✓		
Geometry Box	8									✓	✓

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School Education & Literacy Department (SE&LD)

Government of Sindh