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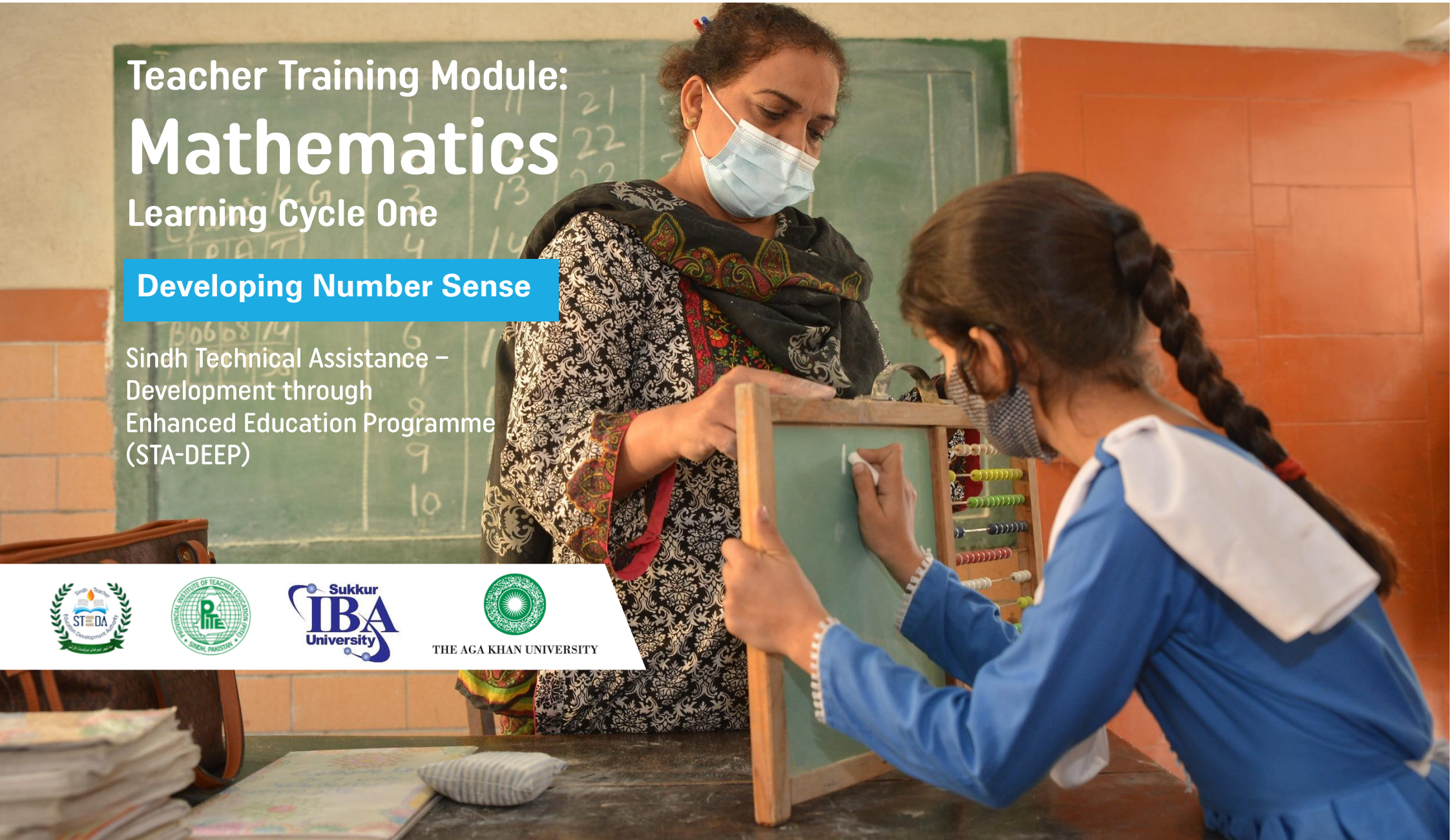
Teacher Training Module:

Mathematics

Learning Cycle One

Developing Number Sense

Sindh Technical Assistance – Development through Enhanced Education Programme (STA-DEEP)



STEDA

Provincial Institute of Teacher Education Sindh, Pakistan

Sukkur IBA University

The Aga Khan University

Introduction and Rationale of the Training

Dear Teachers!

Welcome to the new phase of the Continuous Professional Development (CPD) Program. In the previous phase, we had focused on pedagogical skills that helped you to develop your skills to make classroom more interactive, participative, and joyful for our students. In the new phase, we will continue practicing those pedagogical skills and also learn about the introduced content knowledge and skills in Mathematics, Science, English, Urdu, and Sindhi. As a result, you will be better prepared to deal classroom situation using modern teaching strategies integrated with subject knowledge.

Our vision

Our common goal is to improve the quality of teaching in schools all over Sindh. We want students to become active and collaborative learners, problem solvers, and critical thinkers who approach tasks with creativity and confidence. They are conceptually clear about the subject content and have the skills to link this content with the world around them. To make this possible, we, as teachers, must be better prepared for the classroom demands in pedagogy and the subject content. Moreover, we aim to professionalize these trainings so that the CPD teacher training courses make an impact and substantially change student performance.

Our Teaching Philosophy

The CPD training sessions, including this training, follow a participatory teaching philosophy that engages participants to apply and practice active and collaborative learning, as well as engage in self and peer reflection to become community of practice. The objective is not only to improve the teaching practices but to help you understand the theory of the subject content and the strategies that help students apply the content in daily life with confidence and mastery.

Supporting You

The training module is designed to support you in your classroom teaching. It will introduce you to the subject content and some approaches for use in the classroom. This will make your teaching more manageable and help you grow as a skillful teacher.

Acknowledgement

This module was developed by IBA Sukkur University and Aga Khan University - Institute for Educational Development under the direction of the Provincial Institute of Teacher Education (PITE). It was supported by UNICEF in the scope of the Sindh Technical Assistance Development through Enhanced Education Program (STA-DEEP), funded by the European Union.

We would like to express sincere gratitude to the following contributors:

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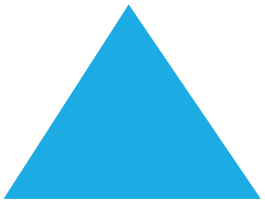


Learning Objectives: By the end of the session, the teachers will be able to:

recognise the attributes of a numerically able child and discuss how students develop their number sense;

use concrete material to represent base 10 numbers and carry out addition and subtraction using those material;

explore multiples and factors through patterns;

unpack meanings of multiplication and division and develop different representations and models of multiplication and division;

connect the ideas discussed in the session with the textbook contents on numbers.

Session Plan

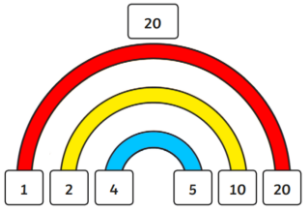
Instructional strategies/activities

Time	Objective/purpose of the activity	Activities/learning experiences	Materials/resources
 15 mins	**Welcome and Introduction** 	1. Tell the teachers that we will use a particular way to introduce ourselves. 2. Ask them to write a number on a piece of paper and think of three characteristics which can help the other fellow teachers recognise the number". 3. Demonstrate the activity starting with yourself, "I am Nusrat. My number has three characteristics. a. It is a multiple of 3. b. It is more than 10 and less than 20. c. Its last digit is 8. d. Shout out my number. 4. Ask the one who shouts first, to now introduce himself/ herself in a similar manner. 5. Continue the activity till all teachers have introduced themselves. 6. Discuss those cases where numbers were not guessed or guessed more than one number. 7. Discuss the advantages of using this strategy in the class. 8. Summarize the activity using the following points;	

		a. By using this strategy teachers will get the opportunity to explore the number which will develop their number sense. b. They will see different ways of explaining the numbers (any other point she/he can add)	
15 mins	**Setting the Ground Rule Using the idea of a bar graph/chart**	1. Ask teachers to write at least three rules of the class on sticky notes (each one on a separate note) and place them on the board/wall". 2. Compile similar rules together with the help of teachers and present them in the form of a bar graph as given in the resource column 3. Discuss the ground rules and seek the consensus of teachers to follow them. 4. Discuss the strategy of setting ground rules and summarize the activity by discussing how mathematics can be integrated into classroom activities and interactions. For example, discussing the percentage or fraction of students absent or present in the class daily; looking at patterns in the calendars; finding angles in the wall clocks; letting them observe shapes around them etc.	
30 mins	**The attribute of a numerically powerful child and how a teacher can help them to develop their numerical strength**	1. Pose the question to the teachers, "In the class, some students are numerically stronger than others. On what basis, do you consider a child is numerically powerful? In other words, how do you identify attributes of a numerically powerful child". 2. Discuss it with the whole class group	Handout 1: Guidelines for Facilitators' Input Handout 2- Principles of Counting

		3. Summarize the activity by using Handout 1 - A Checklist of a Numerically Powerful Child 4. The facilitator will refer to the principle of counting (Handout-2) and ask teachers to share their understanding of each principle. 5. The facilitator will collect their responses and discuss one by one each principle and conclude the activity.	
 30 mins	**Developing an understanding of place value and the formation of Base 10 numbers** 	1. Make groups and ask the teachers to have a brief interactive discussion on the following points: a. The importance of using the manipulative in teaching place values. b. What material should be used in teaching place value? c. What are the pros and cons of some material? d. How the material can be developed, stored, and used, considering economic, logistics, availability, safety and conceptual aspects.) 2. Show pictures of some of the material given in handout 3 and invite teachers to share more ideas and ask them to critically reflect on it (10 min). 3. Invite each group to respond to the above questions and demonstrate an addition/subtraction situation using their preferred material. 4. Highlight the strength and weaknesses of the material using Handout 4: Guidelines for facilitator's Input on the Place value system	Plastic cups; straws; beads (any other relevant material available in the context) Handout 3: List of Resources for Place Value Handout 4: Guidelines for Facilitator's Input on Place Value System
10 mins	**BREAK**		

15 mins	**Representations and Models of Multiplication**	1. Ask teacher to write one story/word problem on multiplication and develop their visual representations. 2. Ask the teachers to review all the illustrations made by their fellow teachers and see how they are different and similar. 3. Give probes based on repeated addition, and array/ area models using Handout 5 "Representations and Model of Multiplication". 4. Help teachers to separate their story problems based on the models they are based on.	Paper slips; glue Handout 5: Representations and Model of Multiplication
20 mins	**Representations and Models of Division** 	1. Brainstorm and ask the teachers, "Is there are any models and representations that could be possible for Division?"  2. Get the teachers' responses and discusses Division as Fair Sharing and Division as repeated subtraction (See Handout 6a). 3. Make groups 4. Ask the teachers to do the activities given in Handout 6b in groups" 5. Take a round to monitor group activity and give feedback.	Paper slips; glue Handout 6a: Guidelines for Facilitators' Input Representations Model of Division Handout 6b: Task Sheet on Model of Division adapted from the below source

20 mins	**Multiples and Factors** 	**Whole Group Discussion** 1. Share the following chart as given in Handout 7: Illustration of Factors and Multiple  2. Ask the following question one at a time; a. What does the patterns of colour show and how the pattern could be continued? b. Which numbers have only one colour or two colours? 3. Ask teachers to share their observation and ask questions.	Handout 7: Illustration of Factors and Multiple 
20 mins	**Alignment with textbooks** 	1. Instruct teachers to work in groups and explore one of the chapters on numbers in the textbooks from Grades 1 to 5, and discuss about teaching these concepts in their classrooms. 2. Give them time to prepare. 3. Invite teachers to share their thoughts and conclude the session.	Textbooks from Grades 1 to 5 (4 sets)
5 mins	**Reflection**	Invite teachers to share their learning experiences during the session with the whole class (refer to reflection questions)	

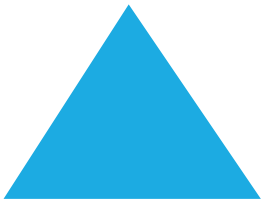
Handout 1: Guidelines for Facilitators' Input

A Checklist of a Numerically Powerful Child

1. An awareness of the relationship between number and quantity What does ten means? two fives? Or ten ones (considering 10 butterflies and 10 elephants are equal in number); five twos? ten steps ahead of 0 on an equally spaced partitioned line or anything else.
2. An understanding of number symbols, vocabulary and meaning Eleven is better to say or 'ten and one'. Have them make a practice they call numbers in terms of place value 2 tens and one for 21 etc.
3. The ability to engage in systematic counting (1-1 correspondence), including notions of cardinality and ordinality. Help children realize that when they count, they can start from any object they will end up with the same result of counting. (See Handout 2- Principle of counting)
4. An awareness of magnitude and comparisons between different magnitudes. (For example, it is better to say 'when a number is multiplied by ten the product is 10 times bigger than that number rather than when a number is multiplied by 10, a zero is added at the end.
5. Competence with simple mathematical operations Students should be able to decompose numbers in varying forms (2+6= 8; 2+4+2=8; 8-6= 2) and begin to recognize how numbers and operations are interconnected. The interconnectedness of mathematics is essential for the correct application of strategies and procedures.
6. Understanding computation strategies and using them appropriately and efficiently.

   e.g., Students should begin to apply formal rules and procedures with an understanding of why a specific procedure or algorithm is appropriate.

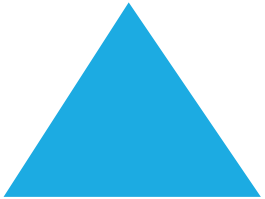
   Students pay more attention to accuracy and precision in their answers as well as demonstrating flexibility with various strategies based on the context of the problem. Estimation becomes a crucial piece in the development of the understanding of efficient and correct strategies.



7. Making sense of numerical and quantitative situations.

   e.g., Students should begin to develop and seek out calculations and relationships that make sense in the context of real-life problems. The "making sense" of mathematics in this category is based on the execution of strategies and algorithms resulting in precise answers that are context dependent. Students' development with the ability to make sense of answers in the real world is the cornerstone of the application of mathematics, the ultimate goal of mathematics teaching.

   For example: **What addition means**:
   Add on
   Two rabbits sat on the grass. Three more rabbit hopped there. How many rabbits are on the grass now? 2+3 =?
   Add together.
   Three red apples and two green apples are on the table. How many apples are on the table? 3+2=?
   What could be the different meanings of multiplications and division?



Principles of Counting

Stable Order Principle

Understanding that the counting sequence stays consistent. It is always 1, 2, 3, 4, 5, 6, 7, etc., not 1, 2, 4, 5, 8.

Order Irrelevance Principle

Understanding that the counting of objects can begin with any object in a set and the total will stay the same.

Conservation Principle

Understanding that the count for a set group of objects stays in the same no matter whether they are spread out or close together.

Abstraction Principle

Understanding that the quantity of five large things is the same count as a quantity of five small things. Or the quantity is the

One-to-One Correspondence Principle

Understanding that each object being counted must be given one count and only one count. It is useful in the early stages for children to actually tag each item being counted and to move an item out of the way as it is counted.

Cardinality Principle

Understanding that the last count of a group of object represents how many are in the group. A child who recounts when asked how many candies are in the set that they just counted, has not understood the cardinality principle.

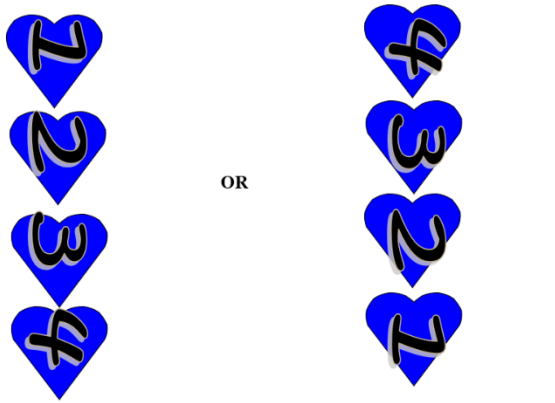
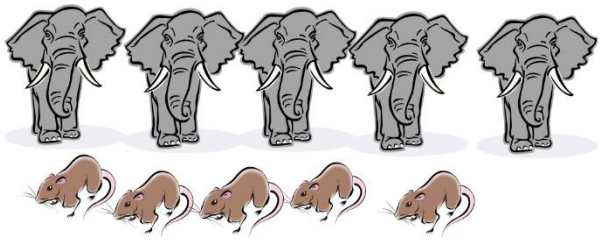
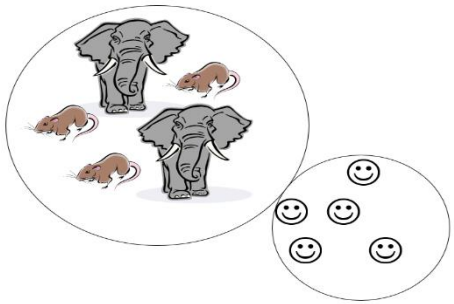
Movement is Magnitude Principle

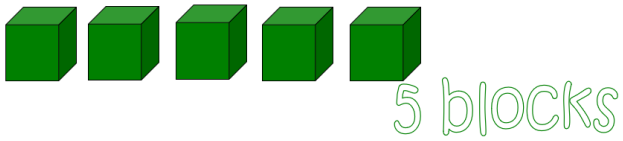
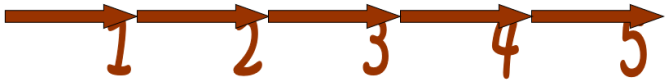
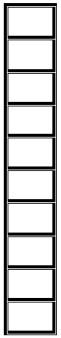
Understanding that as you move up the counting sequence, the quantity increases by one and as you move down or backwards, the quantity decreases by one (or by whatever number you are counting by as in skip counting by 10's, the amount goes up by 10 each time.

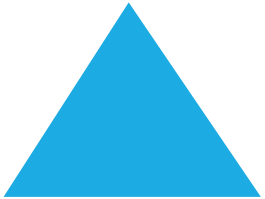
Unitizing Principle

Understanding that in our base ten system objects are grouped into tens when the count exceeds 9 (and into sets of tens when it exceeds 99) and that this is indicated by a 1 in the tens place of a number.

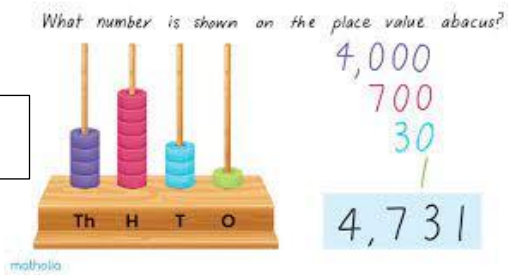
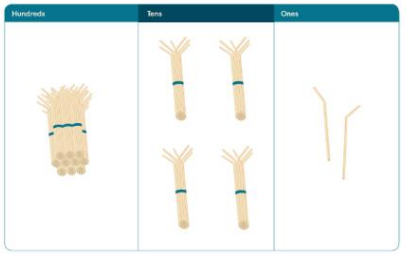
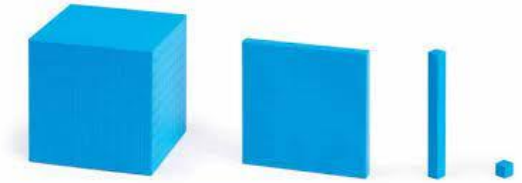
https://www.scribd.com/document/433488644/counting-principles-summarized#

1.	**Stable Order Principle**	1, 2, 3, 4, 5, 6… not 1, 2, 3, 5, 6, 4
2.	**Order Irrelevance Principle**	 Doesn't matter how you count them –there are 4 hearts
3.	**Conservation Principle**	
4.	**Abstraction Principle**	 

5.	**One-to-one Correspondence Principle**	
6.	**Cardinality Principle**	Show Me Five Blocks” 
7.	**Movement is Magnitude Principle**	 
8.	**Utilizing Principle**	  **Hundreds** | **Tens** | **Ones** 1 | 6



Handout 3: List of Resources for Place Value

Beads 	Ball Frame 
Color Pencils 	Sticks 
Cups 	Cubes 

Handout 4: Guidelines for Facilitator's Input on Place Value System

A deep understanding of place value is essential to develop students' fluency in the calculation process. For that notion of "exchange and unitising" is important.

Exchange and Unitizing

Exchange and Unitizing are powerful notions in mathematics for the development of number sense and calculation using all four operations.

Part of developing fluency in calculation processes is the ability to engage with 'one lot of ten' as an entity in its own right and operate with it as though it is one object rather than one object comprised of ten other objects. This capacity is referred to as 'unitizing'.

Children may develop their skill of unitizing through a task in which the children are given the job of packaging seeds for sale. They need to create packs of ten seeds. If they receive an order for say 85, they create 8 sets of ten seeds in envelopes and 5 loose seeds. The children may be left free to explore how to tackle it, scaffolded by appropriate prompts and questions from their teacher. Children start to collect groups of ten seeds and then mark the envelopes in some way to indicate the contents. In doing so they start to develop the capacity to deal with the envelope of ten seeds as one element and this is essential for the development of the capacity to calculate with numbers larger than ten. They develop different ways of recording their numbers and begin to use units of 10 and they are able to say how many packs of ten would be needed to have a collection of 84 seeds for instance, so rounding up to the nearest ten in a meaningful context (Fosnott and Dolk, 2002 see p67 - 75).

References

Fosnott, C.T. & M Dolk (2002) Young Mathematicians at Work: Constructing Addition, Subtraction and Place Value. Portsmouth: Heinemann.

Williams, D. (2012) Mathematics Education is not an Enigma Part 2. Mathematics Teaching. Issue 231 p 6 - 8. Derby: ATM.

Here is a PDF version of this article. Reference: https://nrich.maths.org/10769

Handout 5: Representations and models for multiplication

A representation is a way of showing multiplication. A model for multiplication is slightly more sophisticated and it is comprised of several related representations that all have the same structure. There are two models for multiplication that we'll be comparing and using in this lesson: repeated addition, making equal and array/area.

Repeated Addition is a model for multiplication. Most of the word problems are often on repeated addition representations because they are most clearly abstracted as a repeated addition of numbers:
Direct Modelling with equal sets generally shows a repeated addition of equal amounts. For example: shows 6+6+6+6+6

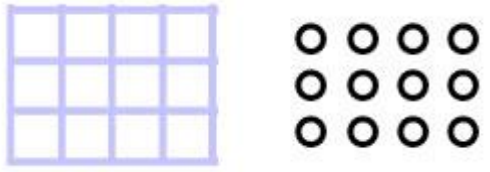
Multiplication bar diagrams show repeated addition of equal amounts represented by equal sized bars:

4	*4*	*4*

It shows 4+4+4

Numerical repeated addition such as 4×7=7+7+7+7 or 4x7= 4+4+4+4+4+4+4 is a representation of a repeated addition interpretation of multiplication

Arrays (Rectangles and Area) is another common and important model for multiplication. An array is a way of arranging the repeated sets so that you can either count along rows or up and down along columns:
These diagrams each show 3 rows of 4 (3×4) and 4 columns of 3 (4×3).



This representation turns out to be really helpful for seeing patterns. It's also a useful representation in higher grades when we study area and start multiplying fractions.
An array model and representation for multiplication help us understand the commutative and distributive properties.

Reference:

https://langfordmath.com/ECEMath/Multiplication/MultModelsText.html#:~:text=There%20are%20two%20models%20for,repeated%20addition%20and%20array%2Farea.&text=Arrays%20(Rectangles%20and%20Area)%20is,and%20important%20model%20for%20multiplication.

Handout 6a: Representations Models for Division

Equal Sharing Model

An equal sharing or Partitive model of division is a situation where the dividend represents the number of objects, and the divisor represents the number of parts (or groups) that the objects will be distributed among. For example, in the problem15÷3, 15 stands for the number of objects and 3 stands for the number of groups. The quotient is 5, which tells us the number of objects that will be placed into each group. For example, if you have 15 balloons and divide them into 3 equal parts, there will be 5 balloons in each group.

Equal Sharing

15÷ 3= 5 is the amount each person gets if 15 items are shared equally among 3 people

15÷ 3= 5

Repeated Subtraction Model

Repeated subtraction is the process of subtracting a number from a large number until the end result is zero. This is a good way to introduce division to smaller grades. For example,

25 - 5 = 20,

20 - 5 = 15,

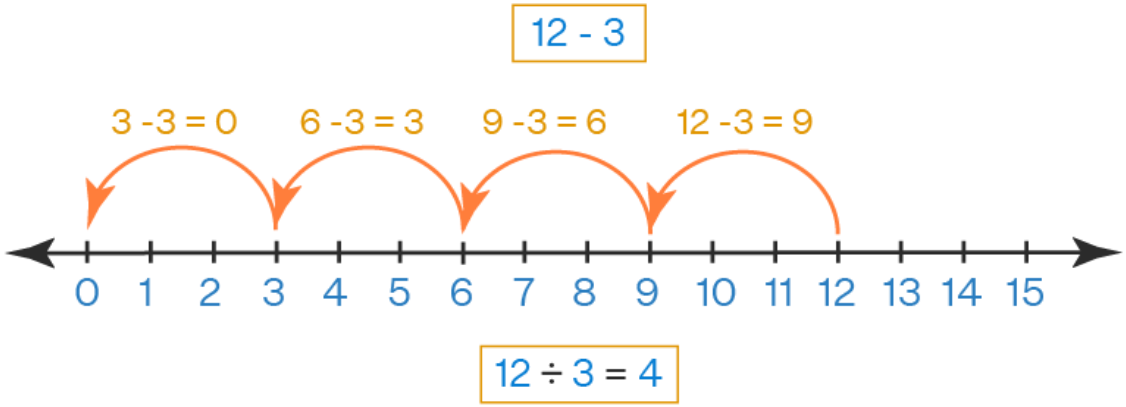
15 - 5 = 10,

10 - 5 = 5,

5 - 5 = 0.

This means the number is subtracted 5 times and can be written in division form as 25 ÷ 5 = 5.

Repeated subtraction can be shown on the number line as given below:



Handout 6b: Task Sheet on Model of Division [1]

Writing Division Scenarios

Try this yourself! Write two scenarios illustrating equal sharing, and two scenarios illustrating repeated subtraction. Challenge someone else to determine which is which and to answer your questions.

Activity One

For each scenario, use pictures, equations, and a sentence to explain the situation. Use as many different equation forms as possible.

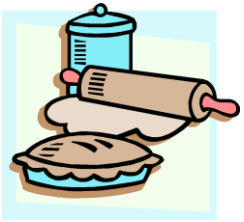
a) 7 children shared 18 coins. How many coins will each child have?

[1] Chrome extensi.on://efaidnbmnnnibpcajpcglclefindmkaj/https://riverbendmath.org/modules/Demystifying_Division/Activities/Demystifying_Division_All_Handouts_And_Keys.pdf

b) 31 biscuits were given to 6 children to share fairly. How many biscuits will each child have?

c) 8 children divide 27 pencils equally. How many should each child take?

Activity Two



a) There are 25 apples in the bag. I want to pack 6 apples in each box. How many boxes can I make? Describe this situation with pictures, equations, and a sentence.

b) I want to put 3 plants in each flower pot. I have 29 plants. How many flowers pots do I need? Describe this situation with pictures, equations, and a sentence.

Activity 3: How Many Stars?

a) There are 34 stars in the box below. How many groups of 2 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

_____

b) Circle groups of 2.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

There are 56 stars in the box below. You will circle groups of 4. Use the partial quotients method to determine how many groups of 4 there will be.

How many groups of 4 do you think you will circle? _____

Explain your reasoning: _____

_____

_____

_____

_____

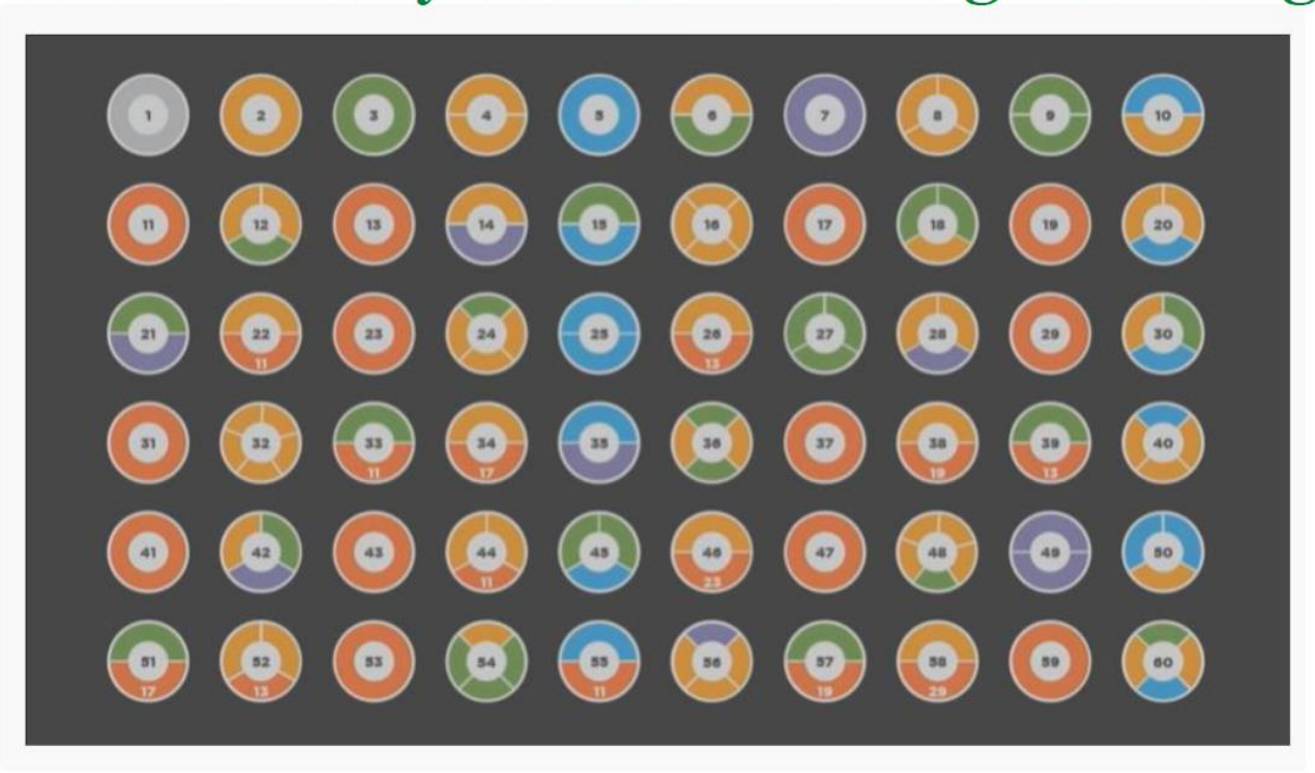
Circle groups of 4.

How many groups did you circle? _____

Write as many equations as you can that describe this situation.

Handout 7: Illustration of Factors and Multiple

Match what you did with the given image



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