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Teacher Training Module:

Mathematics

Learning Cycle Three

Decimal and Percentage

Sindh Technical Assistance – Development through Enhanced Education Programme (STA-DEEP)

STEDA

Provincial Institute of Teacher Education (PITE) Sindh, Pakistan

Sukkur IBA University

The Aga Khan University

Introduction and Rationale of the Training

Dear Teachers!

Welcome to the new phase of the Continuous Professional Development (CPD) Program. In the previous phase, we had focused on pedagogical skills that helped you to develop your skills to make classroom more interactive, participative, and joyful for our students. In the new phase, we will continue practicing those pedagogical skills and also learn about the introduced content knowledge and skills in Mathematics, Science, English, Urdu, and Sindhi. As a result, you will be better prepared to deal classroom situation using modern teaching strategies integrated with subject knowledge.

Our vision

Our common goal is to improve the quality of teaching in schools all over Sindh. We want students to become active and collaborative learners, problem solvers, and critical thinkers who approach tasks with creativity and confidence. They are conceptually clear about the subject content and have the skills to link this content with the world around them. To make this possible, we, as teachers, must be better prepared for the classroom demands in pedagogy and the subject content. Moreover, we aim to professionalize these trainings so that the CPD teacher training courses make an impact and substantially change student performance.

Our Teaching Philosophy

The CPD training sessions, including this training, follow a participatory teaching philosophy that engages participants to apply and practice active and collaborative learning, as well as engage in self and peer reflection to become community of practice. The objective is not only to improve the teaching practices but to help you understand the theory of the subject content and the strategies that help students apply the content in daily life with confidence and mastery.

Supporting You

The training module is designed to support you in your classroom teaching. It will introduce you to the subject content and some approaches for use in the classroom. This will make your teaching more manageable and help you grow as a skillful teacher.

Acknowledgement

This module was developed by IBA Sukkur University and Aga Khan University - Institute for Educational Development under the direction of the Provincial Institute of Teacher Education (PITE). It was supported by UNICEF in the scope of the Sindh Technical Assistance Development through Enhanced Education Program (STA-DEEP), funded by the European Union.

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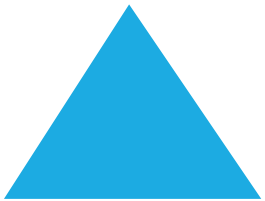
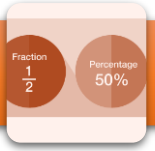


Decimal and Percentage

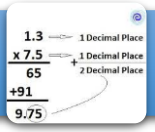
Learning Objectives: By the end of the session, the teachers will be able to:



explore relationship between fractions and decimals;

discuss use of concrete material to represent place value of decimals;

unpack meaning of four operations on decimals and develop representations;

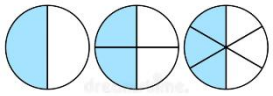
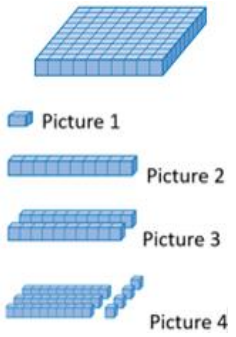


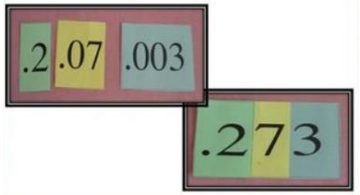
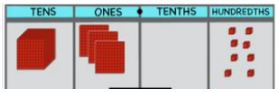
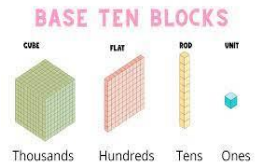
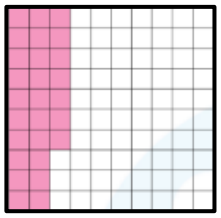
discuss percentages while and connecting it with fractions, and decimals;

connect the ideas discussed in the session with the textbook contents on decimals.

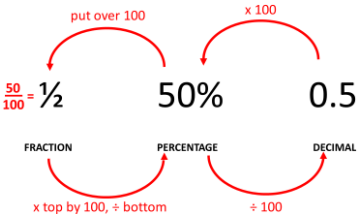
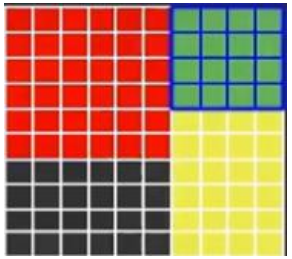
Session Plan

Instructional strategies/activities

Time	Objective/purpose of the activity	Activities/learning experiences	Materials/resources
10 mins	**Welcome and Warm-up Activity** To elicit prior knowledge about fractions. 	**Pair-Activity: 5 mins** 1. Make pairs 2. Give cutouts and Handout-1 to teachers and ask them to do the activity in pairs and give the following instructions, "Place the fractions cutouts in the empty tile so that every row, column and diagonal gives a sum of one whole. To achieve this, you are not allowed to do any written calculation or use a calculator". 3. Take a round to each pair and announce a winner to those pairs who do it correctly. 4. Ask the winners to share their strategy with the class.	1. Handout 1 2. Cut Outs Retrieved from mathcurious.com
20 mins	**Identification of decimal Fraction**  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$	1. Show picture-1 of base ten blocks and ask "Suppose the flat represents one whole. What is each amount worth in pictures 1, 2, 3 and 4? 2. Now show one by one pictures 1, 2, 3 and 4 (given in resource column) and collect responses and write on the board. 3. Introduce the concept of decimals while connecting it with fractions and discuss the following points;	

		a. Those fractions whose denominators are a power of 10, 100, 1000, 10000, and so on are decimals b. Fraction is the relation between a part and a whole. So, in a decimal fraction, the whole is always divided into parts equal to a power of 10 like 10, 100, 1000, and so on 4. Give Handout-2 to each teacher and ask them to perform the given activity. 5. Collect responses from teachers and conclude the activity.	Handout 2
 30 mins	**Developing an understanding of decimals using manipulatives.** 	1. Make groups 2. Ask the teachers to have a brief interactive discussion on the following points: a. The importance of using the manipulative in teaching and learning place values of decimals. b. What material should be used in teaching and learning place value? c. How the material can be developed, stored, and used, considering economic, logistics, availability, safety and conceptual aspects.) 3. Show pictures of some of the material given in the resource column and invite teachers to share more ideas and ask them to critically reflect on it (10 min). 4. Invite each group to respond to the above questions and demonstrate an addition/subtraction situation using their preferred material.	Handout 3   

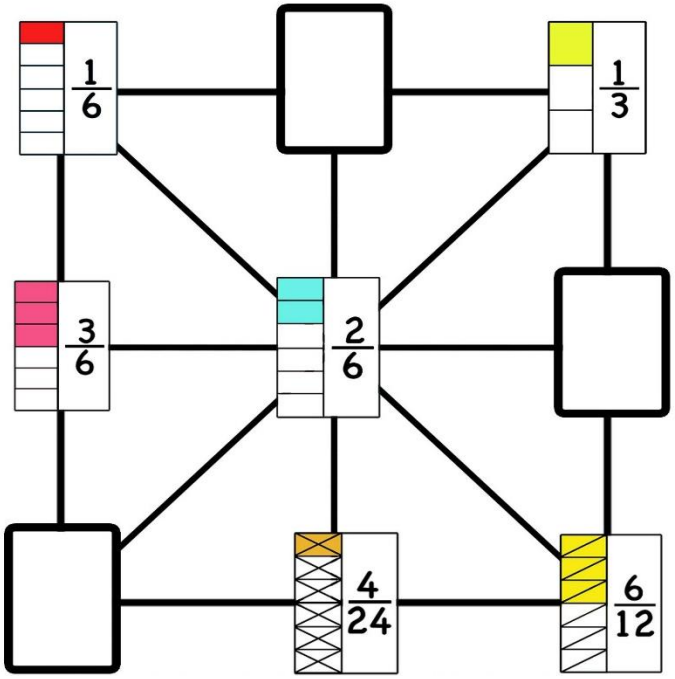
		5. Highlight various concrete materials using Handout 3: Guidelines for facilitator's Input on the Place value system of decimals.	
40 mins	**Activity: Visualization of four operations on decimals using models.** 	1. Divide teachers into groups of four. 2. Write all four following cases on the board. a. **Case 1:** "Using grid paper represents an addition of 0.24 and 0.39". b. **Case 2:** "Subtract 0.21 from 0.83 and represent subtraction using grid paper". c. **Case 3:** Show 0.3 x 0.2 through grid paper. d. **Case 4:** Show 0.8 ÷ 0.2 using grid papers. 3. Discuss and represent each case to teachers through models. (10 min). Refer to Handout-4a for facilitators' input. 4. Give grid papers and colour pencils to each group and ask them "Make a model on grid paper to carry out four operations on decimals using one question from each operator given in the handout 4b". 5. Ask teachers to work in groups and discuss "How visualization models assist in mathematical understanding of four operations on decimals?". (20 mins) 6. Invite one representative from each group to discuss and represent one model to the whole class. Collect random responses from groups for each of the four operations one by one and conclude the activity. (10 mins).	Resource: Grid papers and coloured pencils (blue and black). Handouts 4a for facilitator's Input) Handout 4b

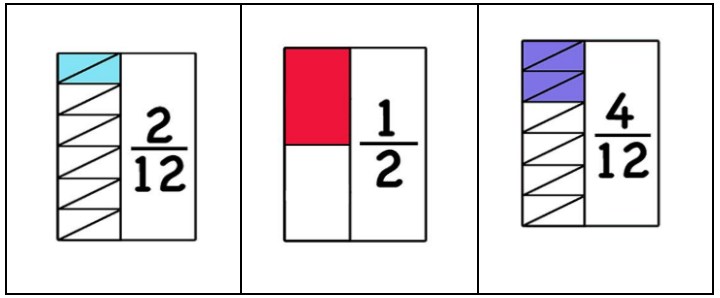
10 mins	BREAK		
20 mins	**Discuss percentages in relation to decimals and fractions** 	1. Show the picture to the teachers and ask following questions: a. What is the fraction of squares shaded in red? b. Can we represent these squares in decimals? c. There are 100 squares in total. What is the number of each colour square out of a hundred? 2. Collect responses from teachers, elaborate on the percentage while connecting it with fractions and decimals. Conclude on the following points; a. Percentage is a special kind of fraction with 100 as the denominator. b. It is the relation between part and whole where the value of "whole" is always taken as 100. c. When we divide the numerator by the common denominator, we get a decimal. For example, 1 divided by 2 equals 0.5. This relates to a percentage in the sense that 0.5 is a half and half is represented in percentages as 50%. So, if 0.5 = 50% and 1/2= 0.5 then 1/2= 50%. Making a fraction into a percent is easy as we simply place the percentage over the whole number 100 and then reduce it. So, 50% becomes 50/100 which is reduced to 1/2. 50% becomes a decimal by then dividing the whole number by a common denominator. 1/2 becomes 2 divided by 1 which equals 0.5 So, the relationship between fractions, decimals and percents is that they are simply different numerical expressions of the same value.	

		Pair-Activity: 1. Give handout-5 to teachers and ask them to cut each square and arrange it in a row so that the value represents the same decimal, fraction and percentages". (10 min) 2. Collect random responses from teachers for each percentage and conclude the activity. (5 min)	Handout 5 Pair of Scissors one for each pair. 
 40 mins	**Alignment with textbooks** 	**Group work** 1. Make five groups 2. Ask the groups to explore one of the chapters on fractions, decimals and percentages in the Grade 4 and 5 textbook and discuss what possible strategies and activities can be used and integrated in teaching and learning decimals and percentages. (20 min) 3. Invite groups to share their thoughts and then sum up the session. 4. Invite groups to share their thoughts with the class. (5min for each group) 5. Sum up the session.	Textbooks from Grades 4 and 5 (five sets)
 05 mins	**Reflection**	Invite teachers to share their learning experiences during the session with the whole class (refer to reflection questions)	

Handout 1

Instructions: Place the fractions cut outs in the empty tile so that every column and diagonal give a sum of one whole.

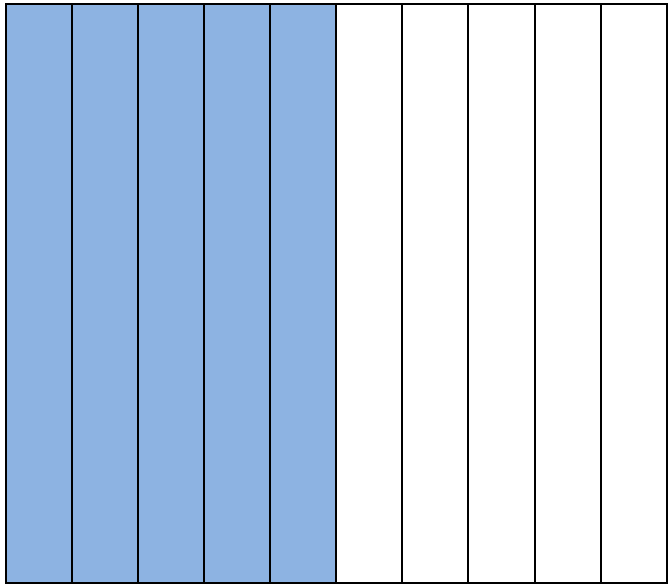




Handout 2

Task 1: Write the fraction and decimal for each given model.

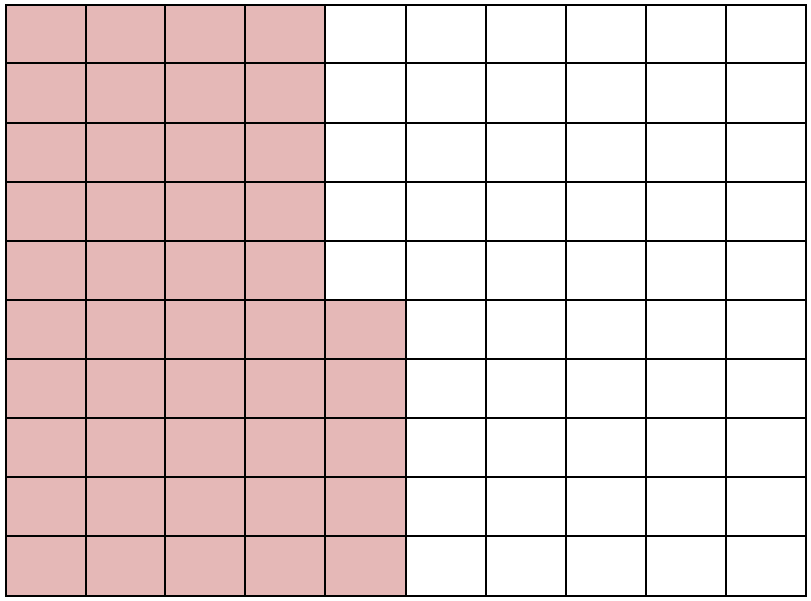
1.



Fraction: _____

Decimal: _____

2.

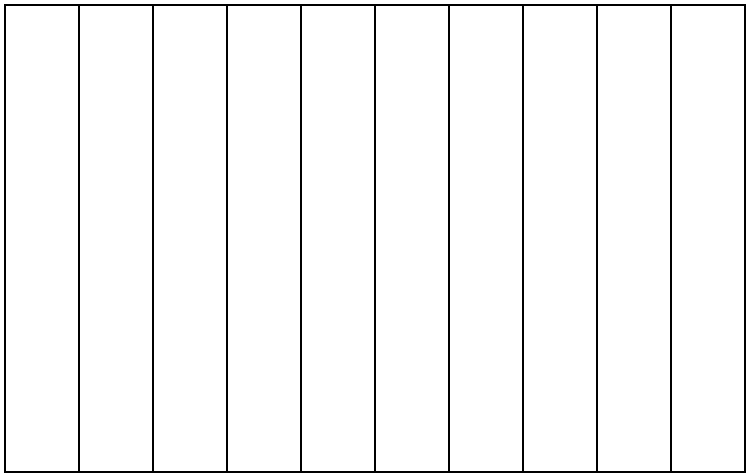


Fraction: _____

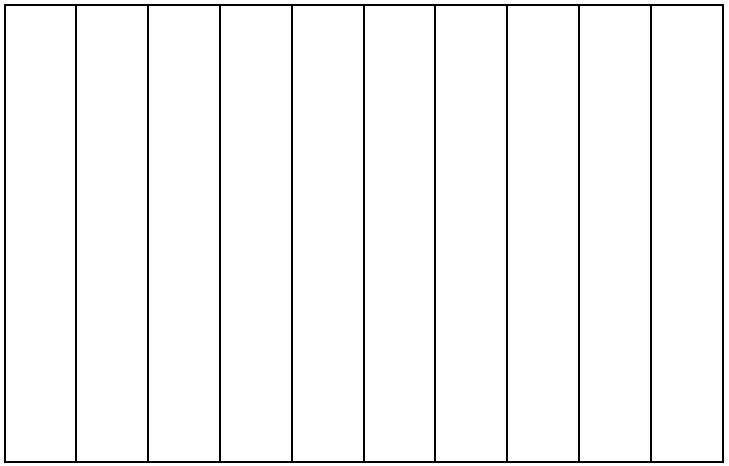
Decimal: _____

Task 2: Use the grid to model the given fraction or decimal.

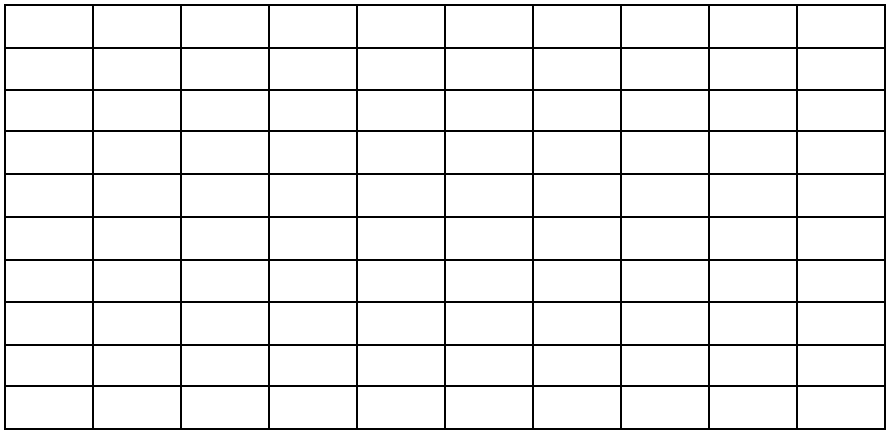
3. Show 0.9 in the table



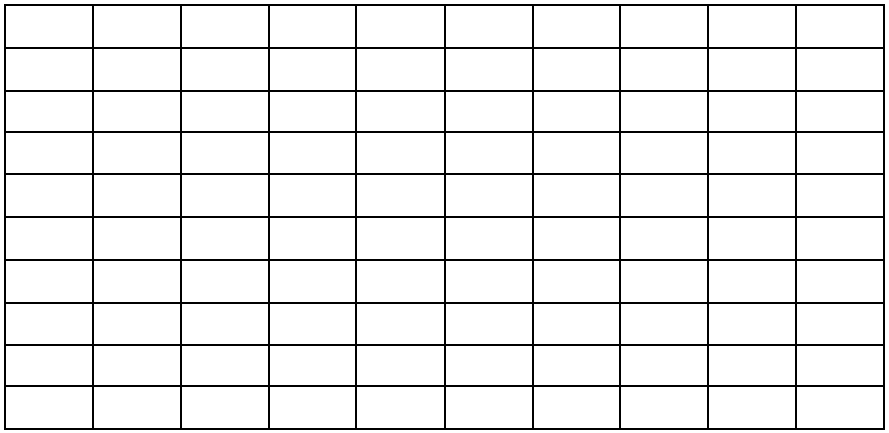
4. Show $\frac{3}{10}$ in the table



5. Show 0.57 in grid

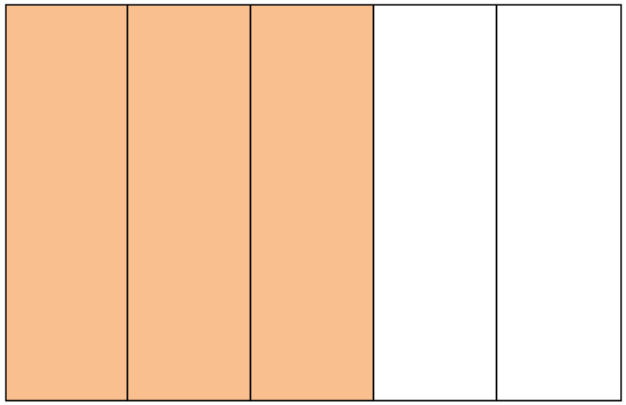


6. Show $\frac{19}{100}$ in grid



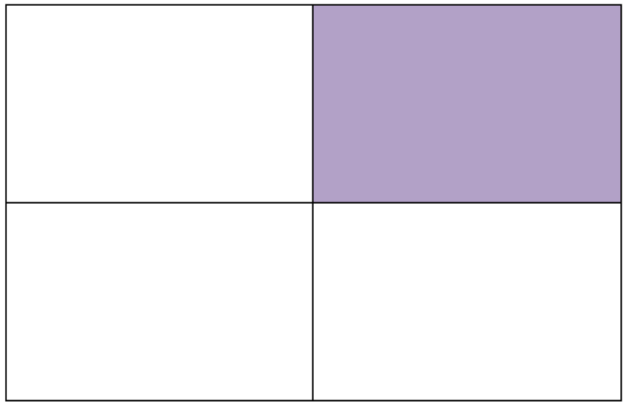
Task 3: Write the fraction and decimal to each model.

1.



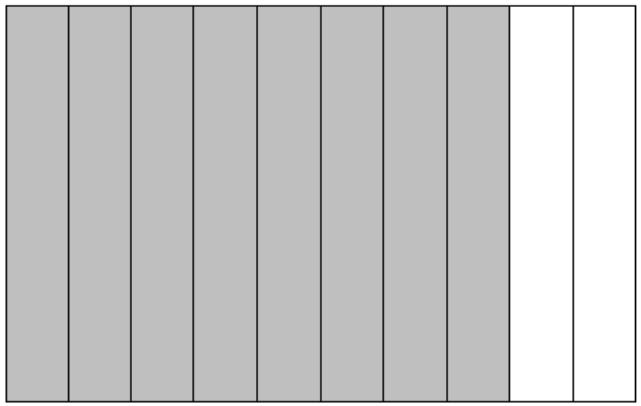
Fraction: _____ Decimal: _____

2.

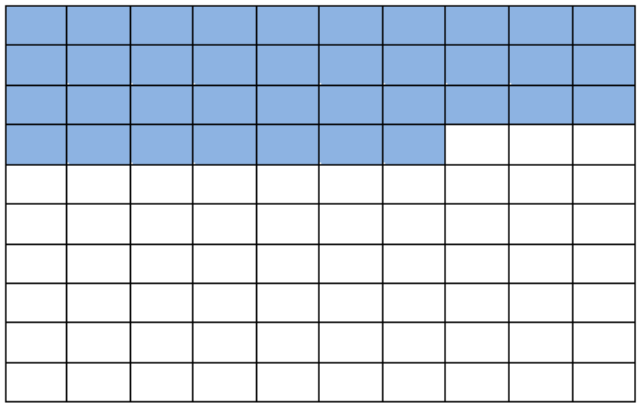


Fraction: _____ Decimal: _____

3.



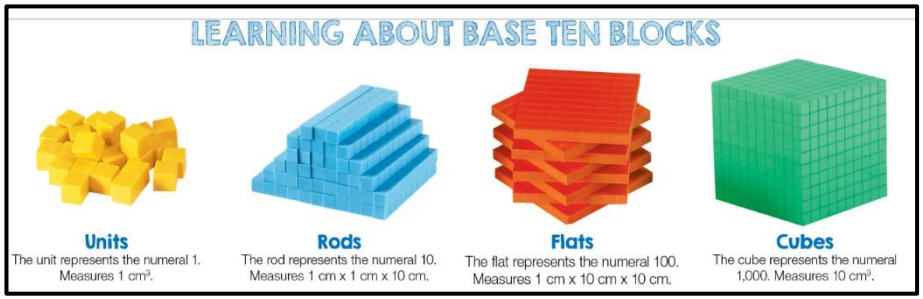
4.



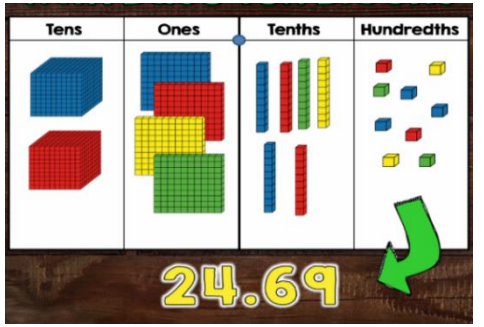
Handout 3: For Facilitators

Introducing Place Value

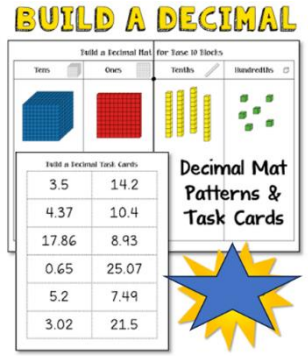
1. **Base ten material:** To start a decimal place value, it's important to start at the concrete level, and base 10 blocks work perfectly because they are sized according to their value. Even 5th graders aren't too old for base 10 blocks. Primary teachers often use them to introduce whole numbers, but base 10 blocks are also effective with upper elementary students when exploring decimals. Kids have to understand that each place to the left is 10 times the size of the place to the right, and base 10 blocks are the best way to explore that concept. When introducing decimals, teachers can use the cubes to represent tens, the flats to represent ones, the rods for tenths, and the units for hundredths.



2. **Place value mat:** Place value mats can help students to remember what place is represented by each model. To create mats, draw the 4-column chart on a large sheet of heavy paper and laminate it. Use a dry-erase marker to draw a decimal point between the ones place and the tenth place. A teacher may divide students into teams of four and give each team one set of bases 10 manipulatives. It's best if each student has a dry-erase board and marker, too. Ask your students to divide up the materials so that one person has the cubes, one has the flats, one has the rods, and one has the units. Introduce each piece and explain how it represents a particular decimal place. The "flat" is equal to 100, which think of it as one whole… maybe one whole cake for a family of mice! If they sliced the cake into 10 parts, each part was 1/10 or 0.1. If they cut those 10 slices into 10 parts, each part was 1/100 or 0.01 of the whole.



Guide your students through an activity "Build a Decimal." Start by writing a decimal in standard form on the board and asking students work to with their teams to "build" that number on the team mat. Students can make up their own numbers. It is important for a teacher to be sure that students represent the number properly on the mat, also ask students to write the word name and expanded form. Expanded form is particularly difficult when representing decimals, and using base 10 manipulatives seem to help illustrate the concept. For example, it's easy to see that the number on the mat can be written as 20 + 4 + 0.6 + 0.09 because students can see each place represented with physical objects. The expanded form of this number could also be expressed as 20 + 4 + 6/10 + 9/100 or completely broken down to 20 + 4 + 6 x (1/10) + 9 x (1/100). All of those ways to express decimals are much more easily viewed when looking at a mat such as the one below. Students can also see that each place to the left is 10 times greater than the one on its right.



As with most math concepts, it's not enough to introduce them with hands-on materials and then move on to the next lesson. Students also have to practice the terminology and work with the concepts until they are fluent in them.

Material to introduce place value can also be made with the help of straws, beads, paper cups, sticks, and cut out pieces of card sheets.

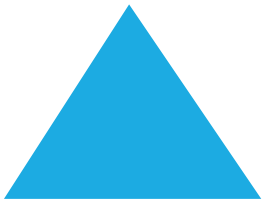
Reference: https://lauracandler.com/introduce-decimals-with-base-ten-blocks/

Handout 4a: Facilitators' Input

The Importance of Visualization in Facilitating Mathematical Understanding:

Mathematics has long been perceived as an abstract and daunting subject, filled with complex equations and obscure concepts. However, the role of visualization in mathematics has transformed our understanding of this discipline, making it more accessible, engaging, and intuitive. Visualization is no longer limited to merely illustrating abstract ideas; it has become a crucial component of mathematical processes, including reasoning, problem-solving, and even proving. By harnessing the power of visual representations, students and mathematicians alike can grasp mathematical concepts more effectively and develop a deeper understanding of the subject. But what exactly is visualization in the context of mathematics? According to Abraham Arcavi, an influential researcher in mathematics education, visualization encompasses the ability, process, and product of creating, interpreting, using, and reflecting upon pictures, images, and diagrams in our minds, on paper, or with technological tools. Its purpose is to depict and communicate information, stimulate thinking, generate novel ideas, and enhance understanding. In essence, visualization enables individuals to transform abstract mathematical ideas into tangible mental or visual representations that facilitate comprehension.

There are numerous examples of how visualization can be applied in mathematics. One of the most recognizable visual representations is the graph, which serves as a powerful tool for communicating mathematical information. Graphs enable us to visualize relationships between variables, identify patterns, and gain insights into mathematical functions. By visually representing data points and their interconnections, graphs provide a clear and concise way to analyze and interpret mathematical concepts. Furthermore, visualization can be employed in mathematical proofs, challenging the traditional notion that proofs must rely solely on rigorous logical reasoning. Visual proofs, such as the visual demonstration of the Pythagorean Theorem, have gained acceptance as legitimate proofs in the mathematical community. These visual demonstrations offer an alternative approach to understanding and verifying mathematical theorems, allowing learners to engage with the subject matter in a more intuitive and visual manner.



Visualizations can also prompt students to reason and explain mathematical concepts using words and symbols. By presenting a visual representation and asking students to analyze the relationships and implications it conveys, educators can encourage critical thinking and deepen students' understanding. This process of reasoning from visual stimuli helps students develop their mathematical thinking skills and enhances their ability to articulate mathematical ideas in multiple forms. Moreover, visualization tasks have the power to stimulate creative thinking and problem-solving abilities. By visualizing possible arrangements, patterns, or spatial configurations, students can approach problem-solving from different perspectives and discover innovative solutions. Activities like counting hexagons, which require students to identify relationships and generate formulas or algebraic expressions, not only strengthen algebraic thinking but also foster a mindset that looks for connections and patterns—a valuable skill in mathematics and beyond. Another significant advantage of visualization is its role in assessing students' mathematical knowledge. By providing visual contexts for assessment, educators can gain insights into students' understanding of mathematical concepts. For instance, asking students to construct test items based on a visual representation not only evaluates their comprehension but also encourages them to apply their knowledge in a meaningful context. This approach provides a more holistic and comprehensive assessment of students' mathematical abilities, going beyond mere regurgitation of formulas or procedures.

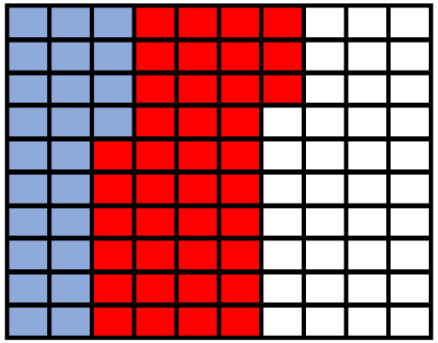
In conclusion, visualization has become a vital tool in the field of mathematics, revolutionizing the way we teach and learn this subject. By enabling the creation and interpretation of visual representations, visualization enhances comprehension, reasoning, problem-solving, and proof techniques. It allows mathematics to transcend its reputation as an abstract and inaccessible discipline, making it more engaging, relatable, and applicable to real-world contexts. As educators and learners continue to embrace the power of visualization, we can unlock new pathways to understanding mathematics and empower individuals to develop a deeper appreciation for the beauty and logic of this fascinating subject.

Reference:

Ronda, E. (2018, October 19). *What is the role of visualization in mathematics?* Mathematics for Teaching. https://math4teaching.com/what-is-the-role-of-visualization-in-mathematics/

Following is the explanation of the visual models for decimals:

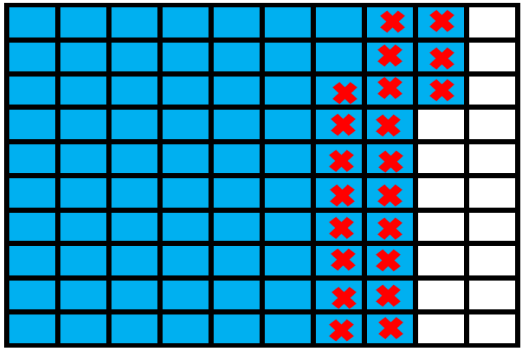
Models with Hundredths Grid



Standard Algorithm

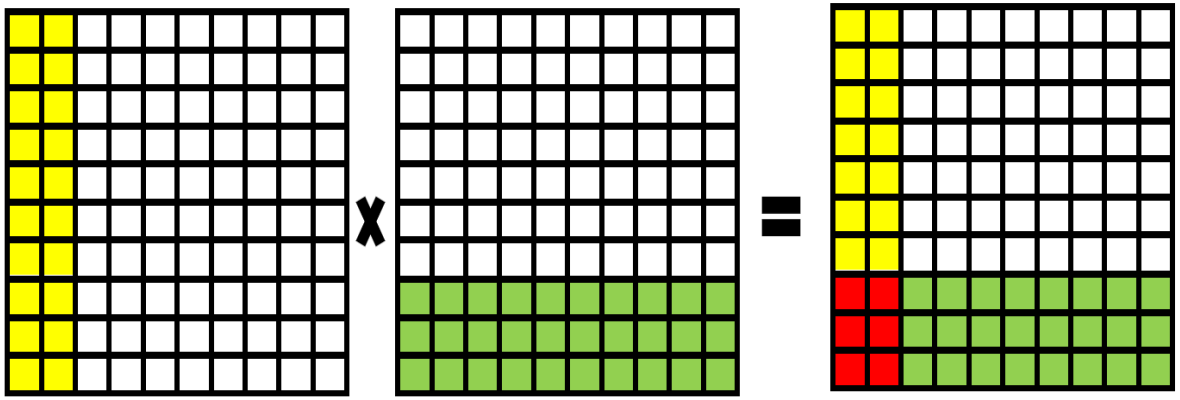
$$\begin{array}{r} 0.24 \\ +\ 0.39 \\ \hline 0.63 \end{array}$$

Models with Hundredths Grid



Standard Algorithm

$$\begin{array}{r} 0.83 \\ -\ 0.21 \\ \hline 0.62 \end{array}$$



3 out of 10 columns are shaded. Therefore, it is 0.3.

2 out of 10 rows are shaded. Therefore, it is 0.2.

6 out of 100 squares are doubled-shaded (0.3 and 0.2). Therefore, it is 0.06.

Four operations on decimals using models:

The model has a hundred squares that it has 10 rows and 10 columns and each square shows a hundredth value

Directions for adding decimals:

First colour the 2 tenths and for hundredth square with blue colour and then we add 3tenths and 9 hundredths square with red colour they all together show 63 hundredth square.

Directions for subtracting decimals:

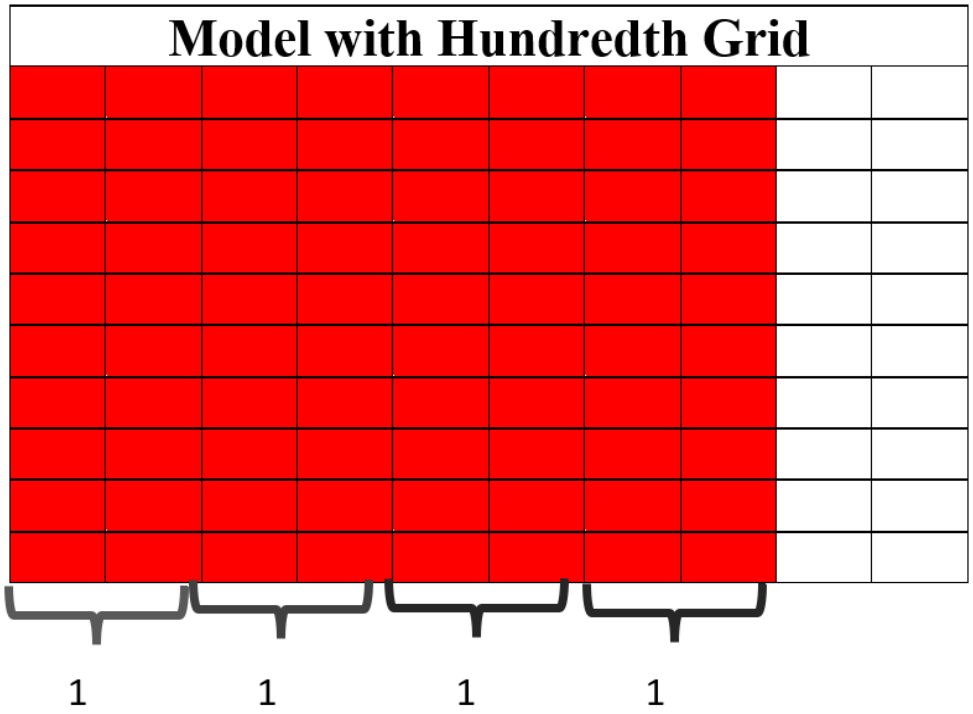
First, shade 83 hundredth squares and then cut 21 hundredth out of them so we have now 62 hundredth shaded squares in this way s/he explained the concept of subtraction.

Directions for multiplying decimals:

First 100 square grid 3 tenths are shaded horizontal and 2 tenths are shaded vertically now double shaded part in third grid show 6 hundredth part of the whole 100 square grid.

Directions for dividing decimals:

Grid 8 column shaded by red colors that represents 8 tenths and then break 8 shaded square in to group of 2 tenths and get 4.



Handout 4b

Show these operations on grid paper

Add

a. 0.5 + 0.8
b. 0.52 + 0.35
c. 0.8 + 2.35
d. 1.3 + 0.25

Subtract

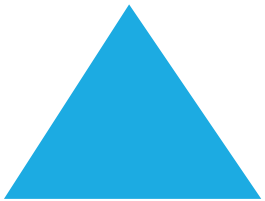
a. 0.8 – 0.4
b. 0.52 – 0.15
c. 1.8 – 0.35
d. 2.8 – 0.65

Multiply

a. 0.4 x 0.2
b. 0.52 x 0.3
c. 0.8 x 0.4
d. 1.3 x 0.5

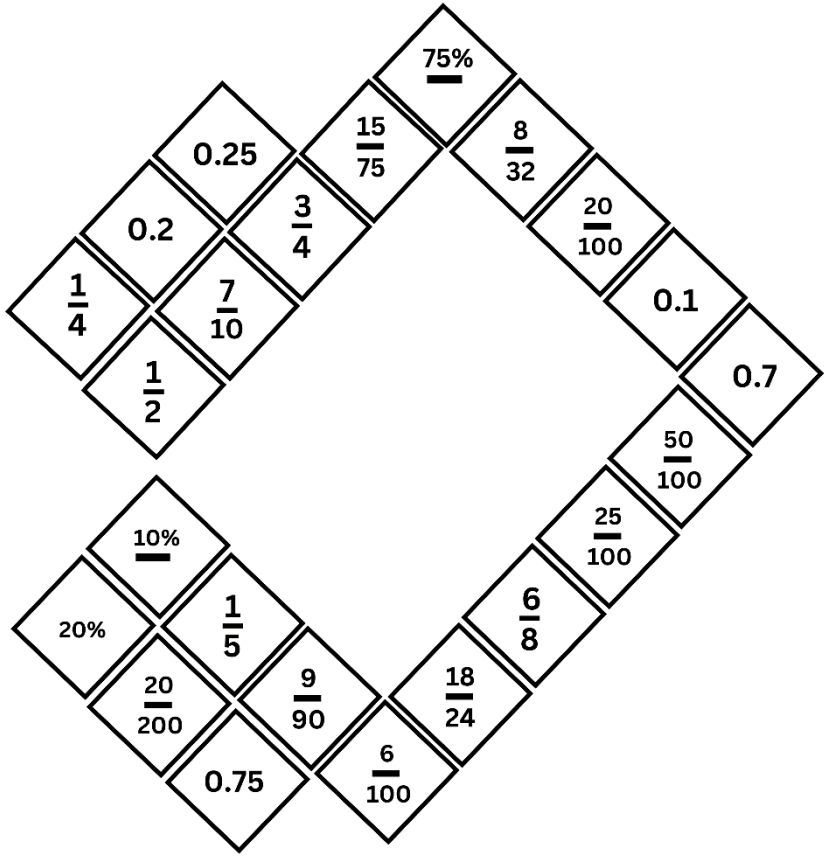
Divide

a. 0.6 ÷ 0.3
b. 0.16 ÷ 0.2
c. 0.15 ÷ 0.5
d. 1.20 ÷ 0.6



Handout 5

Instructions: Cut each square and arrange it in a line so that the value represents the same decimal, fraction and percentages.



Handout: 5 Correct Answers For Facilitator's Reference

Instructions: Cut each square and arrange it in a line so that the value represents the same decimal, fraction and percentage.