Funded by the European Union

Government of Sindh

unicef for every child

Teacher Training Module:

Mathematics

Learning Cycle Five

Introduction to Algebra

Sindh Technical Assistance – Development through Enhanced Education Programme (STA-DEEP)

STEDA

Provincial Institute of Teacher Education Sindh, Pakistan

Sukkur IBA University

The Aga Khan University

Introduction and Rationale of the Training

Dear Teachers!

Welcome to the new phase of the Continuous Professional Development (CPD) Program. In the previous phase, we had focused on pedagogical skills that helped you to develop your skills to make classroom more interactive, participative, and joyful for our students. In the new phase, we will continue practicing those pedagogical skills and also learn about the introduced content knowledge and skills in Mathematics, Science, English, Urdu, and Sindhi. As a result, you will be better prepared to deal classroom situation using modern teaching strategies integrated with subject knowledge.

Our vision

Our common goal is to improve the quality of teaching in schools all over Sindh. We want students to become active and collaborative learners, problem solvers, and critical thinkers who approach tasks with creativity and confidence. They are conceptually clear about the subject content and have the skills to link this content with the world around them. To make this possible, we, as teachers, must be better prepared for the classroom demands in pedagogy and the subject content. Moreover, we aim to professionalize these trainings so that the CPD teacher training courses make an impact and substantially change student performance.

Our Teaching Philosophy

The CPD training sessions, including this training, follow a participatory teaching philosophy that engages participants to apply and practice active and collaborative learning, as well as engage in self and peer reflection to become community of practice. The objective is not only to improve the teaching practices but to help you understand the theory of the subject content and the strategies that help students apply the content in daily life with confidence and mastery.

Supporting You

The training module is designed to support you in your classroom teaching. It will introduce you to the subject content and some approaches for use in the classroom. This will make your teaching more manageable and help you grow as a skillful teacher.

Acknowledgement

This module was developed by IBA Sukkur University and Aga Khan University - Institute for Educational Development under the direction of the Provincial Institute of Teacher Education (PITE). It was supported by UNICEF in the scope of the Sindh Technical Assistance Development through Enhanced Education Program (STA-DEEP), funded by the European Union.

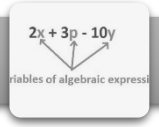
We would like to express sincere gratitude to the following contributors:

Sayed Rasool Bux Shah	Executive Director, Sindh Teachers Education Development Authority (STEDA)
Nusrat Fatima Kalhoro	Director-General Provincial Institute of Teacher Education (PITE)
Tikam Herchandani	Additional Director, Directorate of Teacher Training Institutions Sindh, Hyderabad
Dr. Altaf Hussain Samo	Director Executive Development Center at Sukkur IBA University
Dr. Takbir Ali	Associate Professor and Director Outreach at Aga Khan University Karachi
Shafique Ahmed Memon	Associate Professor, GECE(M), Sukkur)
Dr. Shahid Mughal	Principal GECE(M), Shikarpur)
Dr. Shila Devi	Deputy Director, (STEDA)
Noor Ahmed Khoso	Associate Professor, PITE Sindh Nawabshah
Saima Amir Ali	Module Developer, Agha Khan University (IED), Karachi
Dr. Nusrat Fatima	Module Developer, Agha Khan University (IED), Karachi
Arslan Ahmed	Module Designer, Sukkur IBA University
Abdul Jabbar Shah	Module Designer, Sukkur IBA University
Syed Kamran Shah	Project Manager, Sukkur IBA University
Rabia Batool	Project Manager, Sukkur IBA University
Asif Abrar	Education Specialist, UNICEF
Dr. Salima Begum	Education Officer, UNICEF
Muhammad Zulfiqar Ali	Education Consultant, UNICEF
Aftab Ahmed Nizamani	School Clustering Consultant, UNICEF

Introduction to Algebra

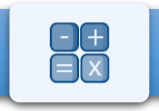
Learning Objectives: By the end of the session, the teachers will be able to:

explore the use of positive and negative numbers in mathematics;



tansform verbal experssions into mathematical expressions and vice versa;

create scenarios where integers are used;



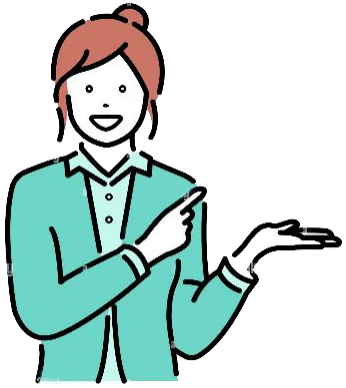
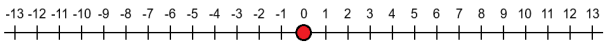
carry out arithmetic operations on integers;

generate number patterns and write generalized (algebraic) expressions for them;

identify different components of the generalized (algebraic) expressions.

Session Plan

Instructional strategies/activities

Time	Objective/purpose of the activity	Activities/learning experiences	Materials/resources
15 mins	**Welcome and Warm-up** To help the participant recognise where positive (+) and negative (-) signs in mathematics are used for types of operations and types of numbers. 	1. Tell the teachers that they will play the Tug of War Game. This game is for two players (teachers). 2. Draw a number line from -13 to 13 on a large piece of paper and find a counter and 1-6 dice to use. 3. Label one player as Positive and the other as Negative. The positive player will move the counter from left to right and the Negative moves the counter from right to left. 4. Place the counter on 0 and start a game.  5. Ask players will take turns throwing the dice and then move the counter that number of places in his/her direction. If the counter reaches -13, the Negative will win. If the counter reaches 13, Positive will win. 6. Play the game two times with different pair of teachers. 7. Ask "What did you learn from this activity?" Note: If possible, draw the number line on the floor and instead of counter players move on the number line	Drawing of the number line from -13 to 13 on a large piece of paper; a counter; and a1-6 dice Reference: The activity adapted from https://nrich.maths.org/5898

		8. Collate teachers' points and conclude with the help of following points; a. Positive (+) and negative (-) signs in mathematics are used for both *types of operations* and *types of numbers* b. The move of players in the activity 'moving left or right represents operations and the point on number lines left or right of zero is the type of number c. In dealing with integers teachers should realise and help their students understand the two different functions of negative and positive signs	
 25 mins	**Creating Scenarios with negative and positive numbers** 	**Group Formation (10 min)** 1. Ask each teacher to pick one chit of paper from the box and explore similar expressions written in different forms. (Handout-1). The teachers who have similar expressions will make a group. **Group Activity (10 min)** 2. Ask the groups, "To further explore integers create a scenario (story or expression) where negative and positive numbers are used. The scenario should reflect negative and positive types of numbers as well as operations". **Sharing Group work (10 min)** 3. The facilitator will invite each group to share their scenarios and ask others to give their feedback.	Sheet of paper with chits on which mathematical expressions are written. (See appendix: Handout-1)

		4. The facilitator will summarize and conclude the activity with the following points; a. Altitude - above Sea level is positive, and below sea level is negative. b. Multi-storey Building: Above ground floor is positive and below ground floor is negative c. Temperature - temperatures below zero are negative. d. Money (Loans) - savings are positive, and loans are negative. e. Science - protons have a positive charge and electrons have a negative charge.	
 30 mins	**Exploration of the rules of adding and subtracting integers.** 	1. Make pairs and give them a pile of black and white buttons/counter, informing teachers that white buttons represent positive integers and black buttons represent negative integers. 2. For each case below, give practice of at least four sums. **<u>Case 1:</u> Adding two positive integers:** 1. Write the example 5 + 3 =? On board and ask teachers, "Represent 5+3 using buttons". 2. The teachers will create one pile of 5 white buttons and another pile of 3 white buttons.	Pile of 20 black and White buttons for each pair. / Class 6 Sindh Textbook Board

3. Give few more sums for practice.

Facilitator's input: When we add integers if the colour (or sign) is the same, we move all the buttons to the end and add them together. In this case, there are 8 white buttons, so the answer is 8 (or positive 8).

Case 2: Adding two negative integers:

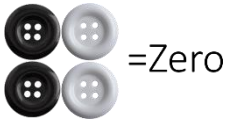
1. Now write an example: (-5) + (-3) =? On board and ask teachers, "Represent (-5) + (-3) = using buttons".
2. The facilitator will ask the teachers to make one pile of 5 black buttons and another pile of 3 black buttons.
3. Help teachers to extract the rule from the activity, "*When we add two integers, if the signs are the same, we keep the sign and add the numbers."*
4. Give some more sums for practice

Facilitator's input: When the colours are all the same, we move all the buttons to the end and add them together. In this case, the answer is -8 because all the buttons are black.

Case 3: Adding two integers with opposite signs:

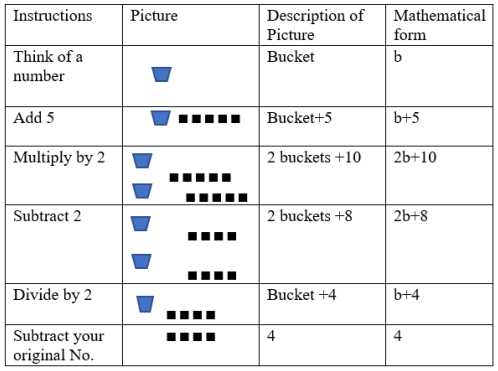
1. Give the example: -5 + (+3) = and explain that the setup is the same, one pile of 5 black buttons and another pile of 3 white buttons. However, since the colours are different, they cannot be added together the way the same colour or sign buttons are added.

		2. Give another example add +1 and -1 and ask what do one black button and one white button make? 3. Introduce the concept of zero pair and refers to adding +1 and -1 and states the result is zero. 4. Now refer to adding +2 and -2 5. Explain that when adding integers with different signs, one black button cancels out one white button until no more zero pairs can be made. The remaining buttons determine the answer and its sign. For example, if there are black buttons left, the answer is negative. 6. Help teachers to extract the rule from the activity and discuss why we traditionally say that plus-minus is minus and a sign of a greater number and extract the rule as, "*When the signs of the two integers are different and you are adding them, take the sign of the larger number and subtract the two numbers".* 7. Give some more sums for practice **<u>Subtracting Integers</u>:** **Case 1: Subtracting a negative integer from a negative integer**. 1. Set up an example: -8 – (-4) = One pile of 8 black buttons and another of 4 black buttons. 2. Tell "When subtracting, the black button directly after the minus sign is replaced with its opposite, a white button. Thus, there will be 8 black buttons and 4 white buttons. This creates a problem of addition. By using additive inverse, we will remove zero pairs,	

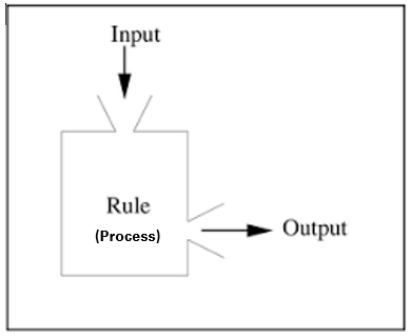


		leaving the number and sign. In this case, the answer is -4". 3. Help teachers to extract the rule from the activity, *"Subtraction of a negative number is equal to the addition of a positive number"* 4. Give some more sums for practice **Case 2: Subtracting a negative integer from a positive integer.** 1. Set up the equation 8 – (-8) =? and 2. Tell teachers "Start with one pile of 8 white buttons, followed by one pile of 8 black buttons. The 8 black buttons are changed to white buttons because subtraction means making the opposite. Now there are 8 white buttons in the first pile and 8 white buttons in the second pile. This becomes an addition problem, resulting in 8 + 8 = 16". 3. Give some more sums for practice 4. Ask few volunteers to summarize the activity.	
 20 mins	**Exploring multiplication of integers through pattern-seeking**	1. Give an unfilled grid handout 2b to each teacher 2. Ask teachers to look at the given grid and find out if they can see any pattern in it. 3. After taking feedback from the teachers, ask teacher to extend the pattern. 4. Help the teachers to extract the following rules of integer multiplication:	Handout-2a for facilitators and Handout- 2b for the teachers Class 6 Sindh Textbook Board

		a. If we multiply two positive integers, the answer will be a positive integer. b. If we multiply a negative integer with a positive integer, the answer will be a negative integer. c. If we multiply two negative integers, the answer will be a positive integer. 5. Also, provide another justification for the first two rules: a. Multiplying a positive by a negative integer: For example, 3 x (-8). This can be written as repeated addition: (-8) + (-8) + (-8) = -24. b. Multiplying a negative by a positive number: (-5) x 4. By applying the commutative property of multiplication, we can rearrange it as 4 x (-5) = (-5) + (-5) + (-5) + (-5) = -20. 6. Ask the teachers to refer to multiplication sums given in STB and perform multiplication of any 3 sums using the understanding of pattern-seeking.	
10 mins	**BREAK**		
 10 mins	**Using pictures for number representations**	1. Present a picture puzzle to the teachers' (Handout 3) and ask teacher to find the hidden values. 2. Engage teachers in a discussion regarding their responses. 3. Following the above activity, the facilitator will then pose a question and ask "If I think of a number, can you represent and write it for me? 4. Collect the responses from the teachers.	Handout-3

<table>
<tr>
<td></td>
<td></td>
<td>5. Conclude the activity and discuss that for any unknown number, we use the English alphabet or any symbol to represent it.</td>
<td></td>
</tr>
<tr>
<td>
30 mins</td>
<td>Understanding of variables</td>
<td>1. Tell the teachers that in the following activity, we will elaborate on the concept of the unknown.
2. Give the following instructions
a. Think of a number
b. Add 5
c. Multiply by 2
d. Subtract 2
e. Divide 4
f. Subtract your original number
g. The number you thought of is?
 
 Holds some number of marbles</td>
<td>Handout-4
Handout-5
Sindh Text Book</td>
</tr>
</table>

Instructions	Picture	Description of Picture	Mathematical form
Think of a number		Bucket	b
Add 5		Bucket+5	b+5
Multiply by 2		2 buckets +10	2b+10
Subtract 2		2 buckets +8	2b+8
Divide by 2		Bucket +4	b+4
Subtract your original No.		4	4

		Activity-in-pair – part a 1. Make pairs 2. Ask teachers to do the think of a number activity in pairs and complete the worksheet given in handout 4. 3. Elaborate on the concept of variables and explain, "a variable is a letter or symbol representing varying values when written in expression". **Activity-in-pair – part b** **1.** Ask teachers to perform the task given in handout 5. **2.** Collect random responses from teachers Ask the teachers "Refer to the Sindh textbook exercise given in algebra and write any two sentences into a mathematical expression"	
 20 mins	**Function Machine** In this activity, the teachers are required to observe the input and output numbers and determine the process or operation that occurs between them.	**Function Machine** 1. Show the following picture of a machine and explain that the machine gives a particular number as *Output* for a number which is placed in the machine as *Input*.  	Handout 6

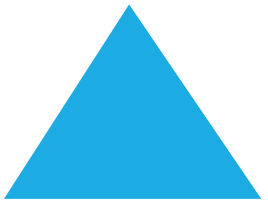
		2. Provide the activity sheet in handout 6 and ask teachers to note down the first three pairs of input and output and figure out the outputs for further inputs. The ones who rightly guess the output would stop and let others to figure out further outputs till the time half of the class would successfully guess". 3. Discuss the responses of teachers and further elaborate on the formation of algebraic expressions. **Group Work** 4. Divide the teachers into groups of four and ask them to play a function machine game. Each member of the group will create their own function machine and keep the rule a secret. One by one, the teachers will show their function machines to the other members of the group, and the remaining teachers will try to guess the rule". 5. After completing the activity, instruct teachers from each group to come to the board and write the expressions, that each member of the group has developed while performing the function machine activity to form a collage 6. Discuss the concept of coefficient, variable, constant and algebraic expressions while referring to the collage	
 10 mins	**Reflection**	Invite teachers to share their learning experiences during the session with the whole class (refer to reflection questions)	

Handout 1

The square of a number is subtracted from the square of another number	The sum of the square of two numbers	$\sqrt{7} \times \sqrt{7}$	Distance between −2 and −9
$a^2 + b^2$	$a^{2-}b^2$	Two is subtracted from 9	The square root of 49
Product of the square of two numbers	The square of the product of two numbers	Twice a number minus 10	The square of the sum of two numbers
a^2b^2	$(ab)^2$	2a-10	$(a + b)^2$
Subtract twice a number from 10	Thrice a number less than ten	Ten less two times a number	The sum of two numbers multiplied by itself
10- 2a	10-3a	Add-10 to twice a number	(a + b) (a + b)

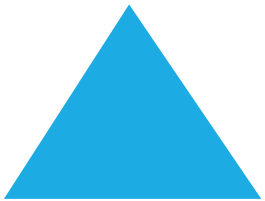
Handout 2a: (For the Facilitator)

	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5	25	20	15	10	5	0	-5	-10	-15	-20	-5
-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
0	0	0	0	0	0	0	0	0	0	0	0
1	-5	-4	-3	-2	-1	0	1	2	3	4	5
2	-10	-8	-6	-4	-2	0	2	4	6	8	10
3	-15	-12	-9	-6	-3	0	3	6	9	12	15
4	-20	-16	-12	-8	-4	0	4	8	12	16	20
5	-25	-20	-15	-10	-5	0	5	10	15	20	25

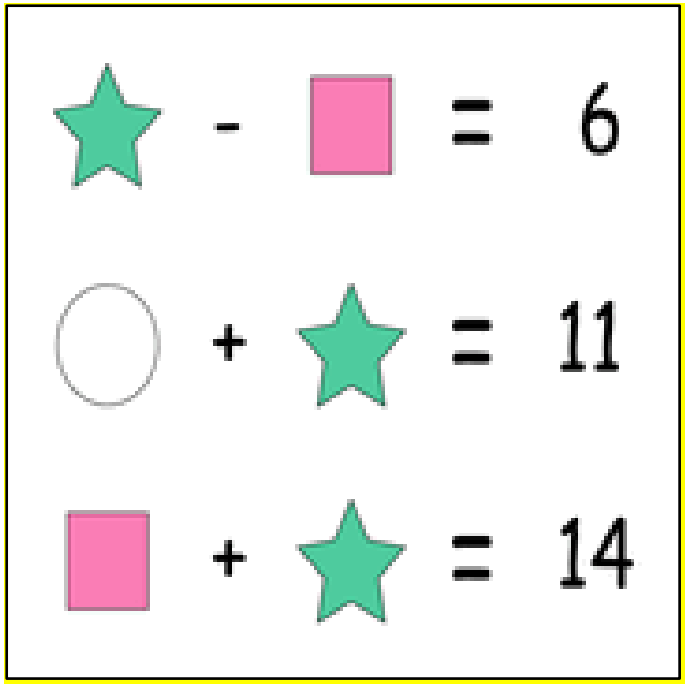


Handout 2b: (For the Teachers)

	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5						0					
-4						0					
-3						0					
-2						0					
-1						0					
0	0	0	0	0	0	0	0	0	0	0	0
1						0	1	2	3	4	5
2						0	2	4	6	8	10
3						0	3	6	9	12	15
4						0	4	8	12	16	20
5						0	5	10	15	20	25

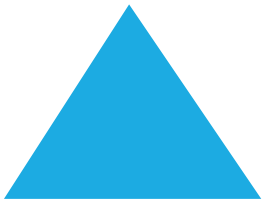


Handout 3



Handout 4

Think of a Number Activity		
Instructions	**Student 1**	**Student 2**
Think of a number		
Add 5		
Multiply by 2		
Subtract 2	48	
Divide by 2		12
Subtract your original number		



Handout 5

Instructions: write the given sentences into a mathematical expression

1. In a garden, I saw 5 squirrels running around. After some time, a few more squirrels joined them. How many squirrels were there in total?

   _____

2. At the zoo, I observed 15 monkeys swinging on a branch. Later, some more monkeys joined them. Can you represent the total number of monkeys?

   _____

3. In a pond, I spotted 8 ducks swimming. After a while, a group of more ducks joined them. Write an expression for the total number of ducks.

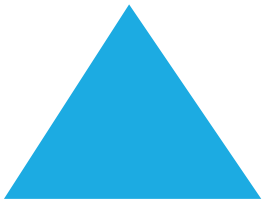
   _____

4. On a farm, there were 20 cows grazing. Some cows moved away, and 10 more cows arrived. Express the total number of cows.

   _____

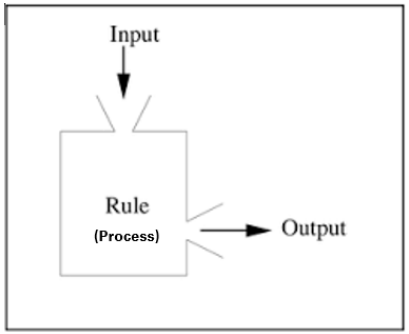
5. In a forest, I observed 18 deer roaming. Later, some more deer appeared. Can you represent the total number of deer?

   _____



Handout 6

Find the rule in the following?



Activity 1

Input	Output
1	7
2	14
3	21
4	28
?	63
50	?

Rule: _____

Activity 2

Input	Output
1	6
2	11
3	16
5	26
?	41
9	?
10	51
?	66

Rule: _____